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What is the Value of Scientific Knowledge? An Application to Global Warming Using the PRICE Model

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Abstract

Governments must cope with the enormous uncertainties about both future climate change as well as the costs and benefits of slowing climate change. This study analyses the value of improved information about a variety of geophysical and economic processes. The value of information is estimated using the "PRICE model" which is a probabilistic extension of earlier models of the economics of global warming. The study uses five different approaches to estimating the value of information about all uncertain parameters and about individual parameters. It is estimated that the value of early information is between $1.5 and $2 billion for each year that resolution of uncertainty is moved toward the present. We estimate that the most important uncertain variables are the damages of climate change and the costs of reducing greenhouse gas emissions. Resolving the uncertainties about these two parameters would contribute 75 percent of the value of improved knowledge.

What is the Value of Scientific Knowledge? An Application to Global Warming Using the PRICE Model [1]

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I. Introduction and Overview

Many scientists believe that humanity is playing Russian roulette with our global environment -- risking irreversible and potentially catastrophic climate changes to eke out a few more percent of global output. Few would argue that mankind is running risky geophysical experiments today, but we know relatively little about the odds of different consequences. Climate-change policies must cope with the enormous uncertainties about both future climate change and about the damages from climate change as well as the costs of preventing that change. For example, our models of both human and natural systems rely upon imperfectly understood geophysical processes, such as the climatic reaction to changing greenhouse gases or the time scale of the reaction. In addition, projections of future emissions, concentrations, and temperature paths depend crucially upon conjectural forces affecting population, productivity growth, and energy efficiency. Moreover, we do not know how fast these uncertainties will narrow or where we would be best advised to apply our research dollars to narrow the uncertainties.

Considerable research today is devoted to improving our understanding of the processes underlying climate change. The U.S. alone is devoting about $1 billion annually to global change research, although much of this is of much broader applicability. Among the major global problems, only war and population receive comparable amounts of scientific effort.

The question that this study addresses is: What is the value of the new knowledge about climate change? If natural and social scientists succeed in improving their understanding, what will be the payoff in terms of improved economic performance? For example, if the uncertainties are resolved in favor of those who argue that global warming

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will be minimal or beneficial, then this knowledge will allow countries to avoid expensive investments in non-carbon energy technologies or in expensive conservation efforts. On the other hand, if the worst fears prove correct, then the globe can mend its ways early so as to prevent later dislocations, famines, or inundations. To the extent that the investments are expensive or the consequences are grave, early information can be extremely valuable.

Another question that can be addressed is the payoffs in different areas. As mentioned above, there are many uncertainties in the climate-change arena. Which are the most important? Where would the payoffs to scientific research be greatest? Are we better off concentrating on the scientific uncertainties about the natural sciences, such as those involved in climate modeling or the carbon cycle? Or are the ecosystem uncertainties crucial? Or do the major uncertainties involve population, technology, or energy systems? We hope to shed some light on the relative importance of different uncertainties.

Before we get into the details, we provide a brief overview of the approach taken here. The question examined is *the value of early knowledge*. That is, we estimate the economic value of resolving the uncertainties about climate change early rather than late. To estimate the value, we embed the uncertainties into a model of climate change and the economy. The model assumes that policy decisions will be made in light of current knowledge as well as economic and technical constraints. If knowledge is extremely limited, then the decisions will reflect the range of possible outcomes and will incorporate a suitable degree of risk aversion given the low-probability but high-consequence events. As knowledge improves, our economic and climate-related decisions can be more focused, and potential waste (from either too little or too much investment in slowing climate change) can be avoided. The value of early knowledge is the improvement in our consumption possibilities that comes from avoiding wasteful decisions because we are too ignorant about future possibilities.

To estimate the value of new knowledge, we proceed in the following steps. We first sketch the PRICE model of the economics of global warming. We then outline the major uncertainties that arise in the construction of the model. Next, we estimate the value of information about *all* the major uncertainties about climate change. Finally, we develop a technique for estimating the value of information for individual variables. We must emphasize at the outset, however, that there is no cut-and-dry technique for estimating these values. Because of the complexity and high dimensionality of the problem, even if we can agree on the proper underlying model of the processes and on the probability distributions of the uncertain variables, we can at this stage do no more than make an approximation of the value of information.

II. Description of the PRICE model

This study examines the impact of uncertainty on policy to combat global warming. It develops a model we call the PRICE model, which is an acronym for a *PRobabilistic Integrated model of Climate and the Economy.* The PRICE model is an extension of the DICE model, which has been used to investigate alternative approaches to global warming.[2] We begin we a brief description of the DICE model because the PRICE model is basically a version of the DICE model that adds another dimension, that of different states of the world. A listing of the equations of the PRICE model along with variable definitions is provided in sections A and B of the Technical Appendix.

The DICE model adopts the standard approach of modern optimal economic growth theory and adds to this both a climate sector and a closed-loop interaction between the climate and the economy. It is an integrated model that incorporates both the dynamics of emissions and impacts and the economic costs of policies to curb emissions. The DICE model asks whether to consume goods and services, invest in productive capital, or slow climate change via reducing greenhouse-gas emissions. The optimal path chosen is one that maximizes an objective function that is the discounted sum of the utilities of per capita consumption. Consumption and investment are constrained by a conventional set of economic relationships (Cobb-Douglas production function, capital-balance equation, and so forth) and by a newly developed set of aggregate geophysical constraints (interrelating economic activity, greenhouse-gas emissions and concentrations, climate change, costs of abatement, and impacts from climate change). Solving the DICE model produces a time sequence of consumption, investment,

[2] See Nordhaus [1991, 1993, 1994].

greenhouse-gas emissions, and carbon taxes which optimizes the objective function.

An analytical description of the methodology is as follows. The DICE model is a "best-guess" or certainty-equivalent model with no uncertainty. It can be written succinctly using the following set of equations:

(1) $\mathbf{Z}(t) = \mathbf{H}(\mathbf{Z}(t - \tau), \mathbf{X}(t - \tau) ; \Gamma_I, \Gamma_{II})$.

where $\mathbf{Z}(t)$ is the vector of endogenous and policy variables (such as output, CO_2 concentrations, carbon taxes, etc.); $\mathbf{X}(t - \tau)$ is a vector of current and lagged exogenous variables (such as population or technology) for lags $\tau = 0, 1, \ldots$. In addition, Γ_I is the set of eight important uncertain parameters that will be examined carefully in this study; and Γ_{II} is the remaining set of uncertain parameters that are omitted from the analysis. **H** is a mapping or vector of implicit functions that represent the discretized Euler equations that are obtained from the optimization of the augmented Ramsey model. The **H** mapping cannot be directly calculated; rather, particular points of the mapping can be obtained by making a model run. Indeed, the major difficulty for the uncertainty analysis arises precisely because the mapping in **H** cannot be directly observed.

The PRICE model takes the DICE model and adds another dimension, which represents the uncertain state of the world. There is obviously an overwhelming number of possible uncertainties to incorporate, so the PRICE model simplifies by aggregating the different uncertain states into five uncertain states of the world. In addition, in considering the uncertainties, we examine only uncertainties about the *eight most important uncertain variables.*

The PRICE model allows for different "states of the world," or SOWs, which represent different realizations of the uncertainties. One state of the world might be one in which population growth is rapid and the damage from climate change is higher than is currently anticipated; another SOW might be one in which there is relatively little warming because of poorly understood feedback mechanisms. Based on a survey of the different uncertainties, we construct different SOWs and attach a probability to each SOW. We then calculate investments and climate-change policy that will maximize the expected value of real income (or "utility").

Finally, for the central purpose of this study, we estimate the maximum attainable real income under different assumptions about the resolution of uncertainties. The *value of early information* is the increase in real income that comes from making decisions based on knowledge about the SOW rather than based on ignorance about the SOW.

Symbolically, we can represent the PRICE model by modifying equation (1) as follows:

(2) $$\mathbf{Z}(t,s) = \mathbf{H}[\mathbf{Z}(t\text{-}\tau,s), \mathbf{X}(t\text{-}\tau,s); \Gamma_{I}(s), \Gamma_{II}], \quad s = 1,..., S.$$

In this extension, there are 5 states of the world, s = 1,...,5. $\mathbf{Z}(t,s)$ is the vector of state-dependent outcome variables, $\mathbf{X}(t\text{-}\tau,s)$ is the vector of state-dependent exogenous variables, $\Gamma_I(s)$ is the set of state-dependent uncertain parameters, and other variables are as in equation (1).

A useful way of describing the value of information is to contrast an act-then-learn approach as opposed to a learn-then-act approach.[3] In a "learn, then act" approach, the decision makers learn about the uncertain parameters, $\Gamma_I(s)$, and then decide on the policy variables. In the "act, then learn" approach, the decision makers are ignorant about the exact state of the world for the first T periods. The policy variables are therefore not state-dependent but are the same across all states of the world for the first T periods. When knowledge about the uncertain variables is gained, the decision maker then knows which state of the world is the relevant one and the policies can at that point be state dependent.[4]

In what follows, we undertake five experiments.

1. Distribution of outcomes. We first estimate the uncertainty of outcomes by doing a Monte Carlo simulation of the DICE model using the full range of uncertain variables, Γ_I. This experiment is useful in providing a rough idea of the extent of uncertainty over time. It is unrealistic as a guide to policy, however, because it assumes that we "learn, then act" -- i.e., that we first learn about all the uncertain variables and then act appropriately.

2. *All variables with expected value states of the world.* A second experiment is to use the PRICE model to estimate the optimal policy under uncertainty with five states of the world. This approach assumes that we can without a major loss of

[3] This terminology has been emphasized in the work of Alan Manne and Richard Richels. See especially Manne and Richels [1990].

[4] In this discussion and in the calculations presented in this paper, we assume that knowledge is an all-or-nothing affair, so decision makers either know nothing or know anything about a given variable.

accuracy aggregate different states of the world into five instead of the millions or quintillions that we would need to capture all the uncertainties. Within this approach, we then can determine the impact of uncertainty on the optimal policy. In addition, we can provide a first estimate of the value of new knowledge or better information.

3. *All variables with random parameters.* A third experiment recognizes that the PRICE model runs in experiment #2 are themselves uncertain. We therefore take different combinations of the uncertain variables to create randomly selected variants of the PRICE model. Using these, we can estimate both the uncertainties about the outcome and about the value of information.

4. *Single variable: expected value approach.* The next set of experiments is designed to estimate the value of knowledge about individual variables. The easiest way to undertake this is to set all but one variable at its expected value and then to examine the effect of individual variables one at a time. Under this approach, we can estimate the value of improved knowledge for a single variable.

5. *Single variable: random value approach.* The final experiment also estimates the value of knowledge about individual variables. This approach takes as the starting point that there is uncertainty about other variables as well by taking a random sample of parameters for other variables. It then estimates the value of perfect information about a single variable.

A detailed description of the modeling of each of the five experiments is contained in the technical appendix.

III. The Value of Information about Global Warming

In this section we present the results from each of the five experiments outlined above. We begin by using the DICE model to develop a distribution of outcomes when eight key parameters are uncertain. Next, we use the PRICE model to execute the remaining four experiments, which allows us to ascertain the impact of uncertainty on the optimal policy for combating global warming and to calculate the value of early information to resolve the uncertainties.

Experiment 1. Distribution of outcomes ("learn then act")

We first outline how we introduce uncertainty into the DICE model and then provide estimates of the importance of the uncertainties on the outcomes. In a first stage, we took the DICE model and did a number of tests to determine the most important uncertain variables. From this stage, we selected eight parameters as the most significant in terms of their effect on near-term policies. The eight uncertain parameters are:[5]

- Population growth (δ_L)
- Productivity growth (δ_A)
- Pure rate of social time preference (ρ)
- Growth of GHG emissions-output ratio (g_σ)
- Damages from climate change (θ_1)
- Climate-CO_2 coefficient (λ)
- Cost of reducing CO_2 emissions (b_1)
- Rate of retention of CO_2 in the atmosphere (β)

In developing a distribution of the uncertain variables, we have discretized each distribution into five quintiles. These distributions are constructed so that the median value equals the expected value and so that the distributions conform to our reading of the scientific consensus about the underlying process. Unfortunately, with five possible values for each of the eight uncertain parameters, there are 5^8 (= 390,625) combinations to be considered. We reduce this to a manageable number by using modified Latin hypercube sampling and taking 625 states of the world. To do this, we use complete enumeration for the three parameters to which the model is most sensitive. These are productivity growth (δ_A), population growth (δ_L), and the pure rate of time preference (ρ). There are 5^3 (=125) possible combinations of these three parameters. For each combination, a variation of Latin hypercube sampling is used to select the other five parameters.[6]

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[5] These parameters are discussed in section D of the Technical Appendix.

[6] Latin hypercube sampling is a method of stratified sampling designed to ensure that all portions of the distribution of all random variable are sampled. The technique is discussed at greater length in section E of the Technical Appendix. It provides more efficient estimators than simple random sampling. (For a technical discussion of Latin hypercube sampling, see Inman and Conover [1980] and McKay, Beckman, and Conover [1979]. A general survey of tools used for the analysis of uncertainty in quantitative risk and policy analysis is provided in Morgan and Henrion [1990].) Latin hypercube sampling works as follows. For a set of *k* random variables, the distribution of each is divided into *N* strata, each of which has probability of 1/*N* of occurring. A random draw is taken

Distributions of the major variables are derived by executing separate runs of the DICE model for each of the 625 states of the world. Table A-2 in the Appendix presents the major results. *The major substantive result here is that the expected values of the control rates and carbon tax are significantly higher than the expected value or best-guess value, indicating that the distribution is skewed upwards.* We show two of the more important results in Figures 1 and 2. Figure 1 shows the estimated distribution of global mean temperature change from 1900 to 2100. The expected value of temperature change in 2100 in the PRICE model is 2.7 degrees C. The 80 percentile range -- that is, the difference between the 90th and the 10th percentile of outcomes -- is 1.9 degrees C. In addition, we have provided bootstrap estimates of the sampling error of these estimates; the estimated error is very modest.[7]

In addition, Figure 2 shows the distribution of current (year 2000) optimal carbon taxes in the PRICE model. The expected value of the carbon tax is $11.64, which is almost three times the median carbon tax. The 90th percentile of carbon taxes is $34, while the 10th percentile is essentially zero. This spread gives an estimate of how uncertain we are about the stringency of the optimal policy today.

Experiment 2. Decisions with uncertainty ("act then learn")

The distribution of outcomes in the last section shows the distribution of possible outcomes in a "learn then act" framework. These results are unrealistic because we must take steps now while ignorant about many of the critical issues; indeed, it may be many decades before we know how natural and human systems will react to increasing concentrations of greenhouse gases. A more realistic approach is one of decision making under uncertainty, which is sometimes called *act, then learn.* This sequence indicates that we must act now even though we will not learn about the true state of the world for many years.

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from each of the N strata for each of the variables, yielding a $k \times N$ matrix of combinations. In our case, since we are working with discrete distributions, each of the five possible values for the uncertain variables is considered one stratum and is sampled once for each set of five states of the world which are paired with the sets of completely enumerated parameters. Thus, for each of the 125 combinations of δ_A, δ_L, and ρ, five possible combinations of the remaining five parameters are chosen. Each of the five possible values of each remaining parameter is used once and only once in each set of five. The resulting 125×5 (= 625) random combinations are used to derive the distributions of the dependent variables.

[7] For a review of bootstrap techniques, see Efron and Tibshirani [1993].

Before describing the approach, we note that it is useful for answering two questions. First, how does the optimal policy change in the face of uncertainty? Should we act as if our best-guess parameters are correct, or should control be more stringent, to prevent catastrophic damages from occurring? The skewness of the results of the Latin hypercube sampling suggests that policy in the face of uncertainty might be more stringent that a best-guess case. Second, and of greater interest to this paper, is the value of scientific knowledge in resolving the uncertainties.

The PRICE model can be used both to show the effect of uncertainty on the optimal control rate and carbon tax and to estimate the value of information. The first step of constructing the PRICE model is to aggregate the large number of potential states of the world into a manageable number -- for this purpose 5 states of the world. These are constructed by averaging over the runs from the Monte Carlo estimates presented in the last section. From this larger sample, we group different parameter values together on the basis of the potential net damage from climate change as measured by the first-period carbon tax. More precisely, the different SOW are constructed by ordering the outcomes according to the stringency of controls in the first period. We then take that ordering and gather the SOW into five groups. The groups are constructed so that the five representative SOW contain 2, 8, 15, 25, and 50 percent of the sample SOW ranked from the most to the least stringent GHG controls. These percentages were chosen to ensure that the extreme outcomes (that is, those with the highest control rates) were well represented.

To model policy with learning, we break time into three phases: (1) the first period is one in which we are assumed to act to maximize the expected value of utility with imperfect information taken into account the distributions of the economic and geophysical parameters; (2) next, at some future date, we learn about the true state of the world--that is, we learn which value from the distribution of economic and geophysical parameters is the true value; and finally (3) for all subsequent periods, we act with *perfect information* about the state of the world and the actual value of the parameters.

To estimate the value of information, we run the PRICE model assuming that the true state of the world is revealed in different years -- the years being now (1995), 2005, 2015, 2025, 2035, and 2045. In each case, we maximize the discounted value of utility contingent on the available information. The earlier the information becomes available, the higher is the expected utility because actions can be better tailored to the costs and damages. The case where information is available at the beginning is the case of perfect information that was analyzed in the last section. By calculating the income gain from earlier information we can calculate the value of early or perfect information.

Figure 3 summarizes the main results of the PRICE model for the emissions-control rate. For each year, the expected value of the emissions-control rate is shown for three possible scenarios -- learning in 1995, 2025, and 2045. Note that when we act in ignorance, the expected value of the control rate is higher than it is in the case of perfect information. Further, in the period after information is revealed (2030 for learning in 2025, and 2050 for learning in 2045) the expected value of the control rate falls relative to the previous period. These results indicate that the optimal policy under uncertainty tends to raise control rates because of the asymmetry in the net damage function.

To better understand the effect of uncertainty on optimal policy, we take a closer look at one scenario. Table 1 presents the optimal carbon taxes and control rates for the various scenarios. Consider, for example, the case where all knowledge is revealed in 2025. Before 2025, the control rate is set to maximize expected utility but is equal in all states of the world because policy makers do not know which state they are in. Policy makers take into account the probability of being in each of the five potential states of the world and choose a control rate which maximizes expected utility given these probabilities. After 2025, policy makers learn about the state of the world and set policy accordingly. The point is illustrated in Figure 4, which shows how the control rate branches after information is becomes available in 2025.

By comparing the results here with the results with perfect information, we see how the control rate and carbon taxes move up sharply when knowledge becomes available for bad states of the world (states 1 and 2), while policies slack off in benign states of the world. *It is interesting to note that, because of the asymmetry in response, it is more likely than not that policy will actually become more relaxed after scientific information improves!*

In addition to showing the effect of uncertainty on the optimal policy, the PRICE model also allows us to make a first estimate of the value of improved information. We proceed by assuming that the uncertainties will be resolved in 2045 if no extra efforts are made to reduce uncertainty. However, society can expend scientific resources to resolve these uncertainties at an earlier date. We look at five possible dates for early learning: 1995 (perfect information), 2005, 2015, 2025, and 2035. We then compare the real income (expected utility in terms of period-one consumption) across the five states of the world in the case with learning in 2045 to the utility in each of the scenarios with earlier learning. In these early-learning scenarios, real income will be higher than in the late information case. The difference in utility between these two cases is the value of information. The value of early information for each early-learning scenario is presented in Table 2.[8] We postpone a discussion of the different estimates to the concluding section of the paper.

Experiment 3. Act then learn with random parameters

The estimates of the value of information in the last section relied on a central set of estimates. This is likely to underestimate the value of information because the parameters on which the uncertainty estimates are based are themselves uncertain. This underestimate occurs because the variance among states of the world for each parameter is less for the set of parameters in the representative scenarios than in the original set of five values for each parameter. Hence, the uncertainties over individual variables are less than in the original distribution presented for each variable. Since there is less variance among the individual parameters, there will be less variance among the optimal policy choices when there is perfect information. As a result, the "mistakes" due to imperfect information will be lessened, and the value of information will be lower than when uncertainty about parameters is introduced.

To determine how important this effect is, we made additional runs of the PRICE model. For each of these, rather than using the representative scenario, a random set of five states of the world was chosen, with the potential parameter values being those from the original distributions. As explained in the last section, only seven parameters were considered uncertain because the discount rate was dropped as an uncertain variable. To choose the values of the uncertain variables, Latin hypercube sampling was employed, so that each potential value of each variable was included in one and only one of the five states of the world. Two hundred sets of five SOW's were randomly drawn, and the PRICE model was run for each set.[9]

[8] One uncertain parameter that poses particular difficulties is the pure rate of social time preference. It is probably not sensible to think that we will learn about future time preference in the same way that we learn about the nature of the climate system. Moreover, the effect of adding states of the world that contain low time preference is basically to weight those states too heavily because they discount future utility so much less. Given these two difficulties, we decided to exclude the rate of time preference from the list of uncertain variables and to concentrate on the other seven. We therefore set the social rate of time preference a 3 percent per year for all runs. Note that dropping the discount rate from the list of uncertain variables reduces the value of information by almost a factor of ten.

[9] A sample trial for this and other experiments is included in section E of the Technical Appendix.

The results of this experiment are shown in Table 3. For each run, the value of information is calculated as in the previous section, but in addition this section provides a distribution of possible values of information because of the uncertainty about the actual outcome. (Bootstrap estimates of the standard errors of the estimates are also included; the calculated standard deviations of the estimates are small.) As expected, allowing for the large number of potential states of the world increases the value of information -- indeed, the estimated value of information is almost doubled.

Experiment 4. Value of Information with Single Variable: Expected Value Approach

The previous two experiments provide us with the value of resolving *all* uncertainties. A more realistic question would concern the relative value of reducing uncertainty in different areas. Is it more important that we resolve natural-science uncertainties, such as the impact of carbon emissions on atmospheric temperature and the impact of global warming on the global ecosystem? Or should our efforts focus on resolving economic uncertainties, such as the impact of population growth or the potential for new technologies to mitigate the impact of the global warming problem? The answers to these questions will provide information about the allocation of limited research budgets concerning global change. We begin in this section by estimating the value of information for individual parameters when all other parameters are set at their expected values (the "expected value approach"); in the next section we calculate the value of information when other variables are uncertain (the "random value approach").

To calculate the value of information in the expected value approach, we proceed by assuming that only one of the parameters is uncertain (this is called the "target parameter") while the other parameters are known. There are for each variable, therefore, five states of the world. In each state of the world, the uncertain parameter for the target parameter varies, and the other parameters are assumed to be equal to their expected values (also equal to their median values). Under these assumptions, the PRICE model can now be represented as:

$$(3) \qquad \mathbf{Z}(t,s) = \mathbf{H}(\mathbf{Z}(t-\tau,s), \mathbf{X}(t-\tau,s); \Gamma_I{}^i(s), \Gamma_I{}^{j\neq i}(3), \Gamma_{II}].$$

As before, $\mathbf{Z}(t,s)$ is the state-dependent vector of outcome variables, and $\mathbf{X}(t-\tau,s)$ is the state dependent vector of exogenous variables. $\Gamma_I{}^i(s)$ is the state-dependent set of uncertain values for parameter *i* (the target parameter). $\Gamma_I{}^{j\neq i}(3)$ represents the remainder of the parameters in question, which are held at their expected value (represented by state of the world s = 3). Γ_{II} represents the rest of the parameters in the PRICE model. As in the earlier experiments, we assume that, barring additional research, uncertainties will be resolved in 2045. The value of information is calculated as the gain in welfare by resolving these uncertainties sooner. The model represented by equation (3) is run for each of the seven unknown parameters with the time of resolution varying for each trial.

The results of this experiment are shown in the first panel of Table 4. This shows that the value of information is greatest for the climate damage (i.e., cost of climate change), for which the value of early information is $31 billion. The next most important variable is the mitigation cost. Surprisingly, the productivity growth rate is relatively unimportant; this is so because in the DICE model productivity growth has offsetting effects on climate-control efforts.

Clearly, the value of information will depend upon the levels and uncertainties of the different variables. While we have taken considerable care in constructing the uncertainty ranges, we recognize that these are also conjectural. Therefore, for each variable we have undertaken two sensitivity experiments of the value of information as a function of the dispersion of the variable and as a function of the expected value of the parameter. In the first experiment, the expected value of each uncertain parameter was doubled in each state of the world holding the standard deviations equal to the base case. In the second experiment, the standard deviations of each uncertain parameter was decreased by one-half while the expected values remained unchanged.

The results of this sensitivity analysis are shown in the bottom two parts of Table 4 and in Figure 5. The results for the dispersion of the uncertain parameters are in line with expectations: the value of information is close to linear in the variance of the parameter. For example, as the standard deviation of the cost of climate change halves, the value of information declines by a factor of 0.238 (that is, decreased by about a factor of four).

By contrast, the sensitivity of the results to the expected values of the parameter is surprising because *the value of information declines as the size of the uncertain parameter increases*. The reason for this surprising result lies in the shape of the policy function which relates the GHG control rate to the uncertain parameters. Denote the policy function as $\mu=f(\Gamma_1,...,\Gamma_7)$, where μ is the GHG control rate and $\Gamma_1,...,\Gamma_7$ are the uncertain parameters. Normalize the parameters so that $\partial\mu/\partial\Gamma_i > 0$. Test runs indicate that $\partial^2\mu/\partial\Gamma_i^2 < 0$ for most of the parameters , indicating that the impact of a higher value of an unfavorable parameter is to increase the control rate but with smaller increments. In such a case, it is likely (but not necessary) that the value of information will also decline as the parameter value increases with constant variance. In other words, a higher parameter value may decrease the policy uncertainty by removing the *favorable* outcomes.

Another paradoxical result concerns the relationship between the value of information and the importance of the variables. While the value of improved information is greatest for climate-change damage and mitigation cost, these are not the parameters to which utility is most sensitive. Real income depends much more on the growth of productivity than on these climate change variables. However, learning about productivity growth will do next to nothing to improve climate-change policy, so the value of information is low for that variable.

Experiment 5. Value of Information with Single Variable: Random Value Approach

A final set of experiments examines the value of information for individual target parameters when the other parameters are taken to be uncertain rather than known. In general, we would expect the value of other parameters to affect the value of information for the target parameter. For example, if the cost of controlling greenhouse emissions is high, it will be more beneficial to avoid doing any unnecessary control. Thus, the value of learning about one variable may depend upon the uncertainty about the others.

The procedure under the random value approach is the following: As in the expected value approach, the distribution of the target parameter is stratified into five different segments or SOWs. The six other parameters, however, are selected randomly from the 5^6 different combinations of values and each of these combinations is assumed to be known with certainty. The interpretation here is that we will learn about all parameters other than the target parameters, but at this point we do not know what the values of the other parameters will be. Hence, we can calculate the value of early knowledge about the target parameter given the full distribution of parameters.

We estimate the value of knowledge for each of the parameters using one hundred draws from the parameters for each of the 7 uncertain parameters. The expected value of information from these runs is presented in Table 5, along with bootstrap estimates of the standard errors. Figure 6 shows the estimates graphically. The estimated value of information is almost twice as high in the random value as compared to the expected value approach; this increase indicates the interaction among the variables. Note that the relative value of the individual variables is virtually identical for the two approaches. Figure 7 shows the distribution of estimates of the value of information for different variables. Clearly, there is a great deal of dispersion in the value of information depending upon the state of the world.

Finally, Table 6 and Figure 8 compare the estimates of the value of information about all the variables by comparing the total values from Tables 2 through 5. From this table we see that the value of information ranges from $45 billion to $108 billion depending upon the exact form of the calculation.

IV. **Conclusions**

In this section we summarize the results and draw the major conclusions for public policy. The prospect of future climate change is an extreme example of a pure global public good, one in which actions of billions of individuals today will have impacts upon the global economy and ecosystem now and for centuries to come. The global public goods character of climate change is exacerbated by the profound uncertainties about all aspects of timing, scope, impacts, damages, and potential costs of abating climate change. Policies for public goods involve not only prevention and adaptation but also improvement in our understanding of the underlying problem. This study asks about the potential benefits of improving our understanding of the climate-change problem. Once the benefits of improved knowledge are estimated, these can be compared with the costs of improving our knowledge so as to reach an appropriate balance.

The approach taken here to estimating the value of information is known as "decision theory." Under this approach, it is recognized that there are uncertainties about the future. However, faced by these uncertainties, policy makers are assumed to optimize real income given the best available knowledge. As knowledge about climate change improves, policy makers can tailor policies more effectively to minimize the net damages. The difference between the expected value of net damages with good information and that with poor information is the value of information.

The results and conclusions drawn here are subject to a number of reservations. First, they depend upon the assumptions and parameters of the actual model used. It will be useful to determine the extent to which the results are replicated in other models of the economics of global warming. Second, the results are highly dependent upon the exact assumptions about the levels and uncertainties of the parameters (see especially the sensitivities shown in Table 4). To a first approximation, the value of information quadratic in the variance of the parameter, so it is straightforward to adjust these results for alternative estimates of levels and variances. Third, the assumption is one that relies on rational decisions by policy makers. Policies are assumed to be set by maximizing the expected value of utility conditional on the state of knowledge. The estimates may be underestimates of the value of knowledge if uncertainty distorts the decision processes (say because policy makers use uncertainty as an excuse either to do nothing or to use the precautionary principle of minimizing regret). Notwithstanding these qualifications, the estimates presented here can provide a rough guide to the value of better information.

The major conclusions of this study are the following. First, a systematic estimate of the uncertainty about future climate change and efficient climate-change policy indicates great uncertainty about the costs and benefits as well as about the appropriate policies. (These results are shown in Figures 1 and 2 and in Table A-2 of the Technical Appendix.) Focusing on current climate-change policies as represented by efficient carbon taxes, we estimate that the 10-90 percentile range of appropriate policies (given perfect information) is close to a factor of 1000 -- ranging between $0.04 per ton and $34 per ton. These correspond to a reduction rate for GHGs of between zero and 24 percent of uncontrolled emissions. Little wonder that arguments about policy are so strenuous in light of the great uncertainties about the efficient policy.

Second, we estimate the value of perfect information -- where that is defined as resolving information at a target date rather than in 2045. It is infeasible to make an exact computation of the value of perfect information. We have therefore presented four different estimates (see Table 6), which indicates that the value of perfect information today (1995) is between $45 and $108 billion. These figures represent the discounted value of the benefit to the global economy of obtaining information 50 years early. Although the relationship is somewhat non-linear, it is approximately correct that moving perfect information forward by one year is worth between $1 and $2 billion.

Third, the major new result in the present paper is an estimate of the value of information about individual variables. Drawing upon estimates of uncertainty from a wide variety of sources, we have estimated the value of better knowledge for seven major parameters. As shown in Table 5 and Figure 5, the order of the value of information is: climate-change damage, cost of reducing emissions, the temperature-CO_2 relationship, population growth, the rate of decarbonization, the atmospheric retention rate of CO_2, and future productivity growth. Roughly speaking, each variable has a value of perfect information equal to the sum of all the variables that come after it. Thus the estimated value of perfect information about climate damage is $55.3 billion while the sum of the values of all the others is $52.3 billion. This result -- which resembles a power law -- has no analytical foundation but is an interesting curiosum.

Fourth, it is interesting to note that new information is most valuable for parameters that are most directly related to climate-change policy -- specifically the damages from climate change and the mitigation cost of reducing GHG emissions. The value of information is least important for variables that are quite remote from climate change policy, for example, information relating to productivity growth or the carbon cycle. (See Tables 4 and 5 for these estimates.) The two directly related climate-change variables (damages and mitigation cost) account for about three-quarters of the value of improved information.

Finally, how do our estimates of the value of information compare with research investments in the economics of global change? Most research on global change has concentrated on uncertainties about the natural sciences -- particularly those involving climate modeling and the carbon cycle. For the federal global-change budget, approximately 95 percent lies in the natural sciences, with about 5 percent being in the social and behavior sciences. This allocation contrasts sharply with the value of information, as can be seen in Tables 4 and 5. According to our estimates, the uncertainties involving natural sciences comprise about 15 percent of the total quantified uncertainties, while those involving behavioral and social sciences account for about 85 percent of the value of better knowledge. One explanation of the imbalance is that it might prove much more difficult to resolve uncertainties for social and behavioral sciences than for natural sciences. If this were the case, then an attempt to balance incremental costs and benefits might tilt the research funding toward the natural sciences. A more plausible source of the difference in allocations, however, is that the social and behavioral sciences are latecomers to the global-change research table and are simply living off the crumbs.

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Technical Appendix. Details of Calculations for PRICE Model [PRIC0219.APP]

This appendix gives the details of the PRICE model. The sections are (A) variable definitions in the PRICE model; (B) the complete equation listing; (C) a description of the five major experiments in the text, (D) a derivation of the distribution of the uncertain variables, and (E) the details of the Latin hypercube sampling.

A. Variables

The variables are as follows. We have omitted certain obvious equations such as the capital balance equation and the definition of per capita consumption. In the listing, t always refers to time (t = 1990, 2000, ...) while s refers to the state of the world. Those variables or parameters followed by a state-of-the-world index are state-dependent.

Exogenous Variables (generically represented by X(t,s)

A(t,s) = level of technology
P(t,s) = population at time t, also equal to labor inputs
O(t) = forcings of exogenous greenhouse gases
s = uncertain state of the world (SOW)
t = time

Parameters [generically represented by $\Gamma(s)$]

α = elasticity of marginal utility of consumption
$b_1(s)$, b_2 = parameters of emissions-reduction cost function
$\beta(s)$ = marginal atmospheric retention ratio of CO_2 emissions
γ = elasticity of output with respect to capital
δ_K = rate of depreciation of the capital stock
$\lambda(s)$ = feedback parameter in climate model (inverse to temperature-sensitivity coefficient)
p(s) = probability of the state of the world
$\rho(s)$ = pure rate of social time preference
$\sigma(t,s)$ = CO_2 emissions/output ratio
ξ_i = parameters of climate equation and carbon cycle
$\theta_1(s)$, θ_2 = parameters of climate damage function

Endogenous Variables [generically represented by Z(t,s)]

C(t,s) = total consumption
c(t,s) = per capita consumption
D(t,s) = damage from greenhouse warming
E(t,s) = CO_2 emissions
F(t,s) = radiative forcing from all greenhouse gas concentrations
$\Omega(t,s)$ = output scaling factor due to emissions controls and to damages from climate change
K(t,s) = capital stock
M(t,s) = mass of CO_2 in atmosphere (deviation from preindustrial level)

Q(t,s) = gross national or regional product
T(t,s) = atmospheric temperature relative to base period
T*(t,s) = deep ocean temperature relative to base period
U(t,s) = U[c(t,s),P(t,s),s] = utility of consumption
V = social welfare function determined by country consumption levels
Y(t,s) = gross world product (net of climate damage and mitigation costs)

Policy Variables [generically represented by $\pi(t,s)$]

I(t,s) = gross investment
μ(t,s) = rate of emissions reduction

B. Equations of Basic PRICE Model

$$(B.1) \qquad \underset{c(t,s)}{Max} \quad V = \sum_{t=0}^{T} \frac{p(s)\, U[c(t,s),P(t,s),s]}{[1+\rho(s)]^t}$$

subject to

$$(B.2) \qquad Q(t,s) = A(t,s)K(t,s)^{\gamma}P(t,s)^{1-\gamma}$$

$$(B.3) \qquad Y(t,s) = \Omega(t,s)Q(t,s)$$

$$(B.4) \qquad C(t,s) = Y(t,s) - I(t,s)$$

$$(B.5) \qquad E(t,s) = [1-\mu(t,s)]\sigma(t,s)Q(t,s), \qquad 0 \le \mu(t,s) \le 1.$$

$$(B.6) \qquad M(t,s) = \beta(s)E(t,s) + (1-\xi_M)\,M(t-1,s)$$

$$(B.7) \qquad T(t,s) = T(t-1,s) + \xi_1[F(t,s) - \lambda T(t-1,s)] - \xi_2[T(t-1,s) - T*(t-1,s)]$$

$$(B.8) \qquad T*(t,s) = T*(t-1,s) + \frac{T(t-1,s) - T*(t-1,s)}{\xi_3}$$

$$(B.9) \qquad F(t,s) = \frac{4.1 \ \log[M(t,s)/M(0,s)]}{\log(2)} + O(t)$$

$$(B.10) \qquad \Omega(t,s) = \frac{1 - b_1(s)\,\mu(t,s)^{b_2}}{1 + \theta_1(s)\,T(t,s)^{\theta_2}}.$$

C. Equations Used for Experiments

The calculations of the value of information start with the basic PRICE model and then impose certain constraints on actions. For this discussion, we can analyze the model in terms of the following succinct representation:

$$(C.1.K) \qquad \underset{\pi(t,s)}{Max} \;\; E(V) = \sum_{t=0}^{T} \frac{p(s)\,U[c(t,s),P(t,s),s]}{[1+\rho(s)]^t}$$

subject to

$$(C.2.K) \qquad Z(t,s) = H[Z(t-\tau,s),X(t-\tau,s);\beta_I(s),\beta_{II}] \qquad s=1,\ldots,S;\ \tau=0,\ldots.$$

In these equations, K = 1,...,5 represents the experiment number below. The variables Z(t,s) are the endogenous and the policy variables, those labeled X(t-τ,s) are the exogenous variables, and the π(t,s) are policy variables. The notation is the equation (k, m, n) represents equations for section k, equation number m, and experiment number n. The equation set in *(C.2.K)* represents the constraint equations (2) through (10) listed in section (B) above as modified for the particular experiment. In the five subsections that follow, we describe each of the five experiments.

Experiment 1. Distribution of outcomes ("learn then act")

In the approach of "learn, then act," it is assumed that decision makers know the state of the world before taking any decisions. There are 625 randomly selected SOWs, selected by Monte Carlo simulation using Latin hypercube sampling. All decisions are state-dependent. Hence, the equations are:

$$(C.1.1) \qquad \underset{u(t,s)}{Max} \;\; E(V) = \sum_{t=0}^{T} \frac{p(s)\,U[c(t,s),P(t,s),s]}{[1+\rho(s)]^t}$$

subject to

$$(C.2.1) \qquad Z(t,s) = H[Z(t-\tau,s);\ \beta_I(s),\beta_{II}] \qquad s=1,\ldots,625;\ \tau=0,1,\ldots.$$

Experiment 2. Decisions with uncertainty ("act then learn")

In the second experiment, the SOWs are aggregated into 5 representative SOWs. Learning takes place in year T $\geq$ 1995. For those periods after learning takes place, the equations (C.1.2) and (C.2.2) are identical to equations (C.1.1) and (C.2.1) in experiment 1. For earlier period, the policies are no longer state dependent and are replaced by:

$$(C.3.2) \qquad u(t,s) = u(t) \qquad s = 1,...,5;\ t<T$$

Equation (C.3.2) states that for periods until uncertainties are resolved, the control variables must be identical across all states of the world.

The major differences among experiments 2 through 5 comprises the selection of the uncertain parameters, $\Gamma_I(s)$. In experiment 2, these are chosen to be "representative scenarios." The technique for choice is as follows: We begin by taking the 625 SOWs from the Monte Carlo sample in experiment 1 and rank them in terms of the economic importance of climate change as measured by the period-one carbon tax. We then take that ordering and gather the SOWs into five groups. The groups are constructed so that the five representative SOW contain 2, 8, 15, 25, and 50 percent of the sample SOW ranked from the most to the least stringent GHG controls. These percentage were chosen to ensure that the extreme outcomes (those with the highest control rates) were well represented, while that half of the sample SOW with but modest GHG control requirements were aggregated into a single aggregate SOW. Finally, we average the parameters in each group to get the representative sample. Hence, SOW1 is the average of the top 2 percent (=.02 $\times$ 625= 12 runs from the sample); SOW2 is the average of the top 8 percent (=.08 $\times$ 625= 50 runs); and so forth. The parameters are constructed so that the average value of the parameters is (except for sampling error) equal to the expected value of the parameter in our original estimates. Hence, the parameters are given by:

$$(C.4.2) \qquad \beta_I(s) = E[\beta_I(s), \textit{ where s is aggregated out of the Monte Carlo simulation }]$$

Experiment 3. Act then learn with random parameters

Experiment 3 resembles experiment 2 in that there are 5 states of the world and the expected value of information is calculated for different dates of learning (T). In experiment 3, the parameters for the 5 SOWs are chosen randomly from the underlying distribution of parameters, whereas in experiment 2 the parameters were chosen to be representative as explained above. Hence, in this experiment, equations (C.1.3), (C.2.3), and (C.3.3) are as in experiment 2, while the fourth equation is:

$(C.4.3)\quad \beta_I(s)\quad$ *drawn out of underlying distribution for* β_I

Experiment 4. Value of Information with Single Variable: Expected Value Approach

Experiment 4 examines the value of information for individual parameters. For this experiment, the "target parameter" (Γ_i) is examined, while the remainder of the parameters are set at their expected values. Hence equations (C.1.4) through (C.3.4) are as in experiments 2 through 5 while the equations for the parameters are as follows:

$(C.4.4)\quad \beta_I^i(s)\quad$ *drawn out of underlying distribution for* $\beta_I^i(s)$

$(C.5.4)\quad \beta_I^{j*i}(s)\quad$ *equal expected value of* β_I^{j*i}

Experiment 5. Value of Information with Single Variable: Random Value Approach

Experiment 5 is identical to experiment 4 except that the values of the non-target parameters are randomly selected rather than set at their expected value. Hence all, equations are identical to those in experiment 4 except for (C.5.4), which is replaced by:

$(C.5.5)\quad \beta_I^{j*i}(s)\quad$ *drawn out of underlying distribution for* β_I^{j*i}

D. ***Distribution of Uncertain Parameters***

The uncertain parameters were listed in the body of the paper and will be discussed in detail in this section. The basic results are shown in Table A-1. Part I of table A-1 shows the distribution of the uncertain parameters for each of the five states of the world. Part II shows the distribution of parameters which make up the representative scenarios of experiment 2. The tables show the expected value, the median, and the standard deviation for the eight uncertain parameters. In addition, these show the quintile values for each of the eight uncertain variables. Note that in experiments 2 through 5, the rate of time preferences is not varied.

This section provides a sketch of the distribution. In most cases, the estimates are drawn from Nordhaus [1994], and interested readers are referred to that source for a fuller discussion. Note that it is relatively easy to adjust the results for different assumptions about the means and standard deviations of the estimates. If researchers prefer different distributions of the uncertain parameters (within the general structure of the DICE model), then the correction factors shown in Table 4 will show the "multiplier" of the value of information for the first two moments of the distribution.

The derivation of the distributions of the uncertain variables varies considerably because of the nature of the underlying data. The variables fall into three general categories: (1) Variables that have an extensive history but which are evolutionary; this class includes population growth and productivity growth, for which we have historical data for many decades but where the structure is not well enough understood to allow secure prediction of future values. (2) Variables that are scientific relationships which are invariant over time but whose values are uncertain because the appropriate experiments cannot be made; this includes important relationships such as the carbon cycle and the temperature-GHG parameters. (3) Finally are relationships that are both evolutionary and for which there are no adequate historical data and which therefore pose the most severe issues for estimating the distribution of uncertain values. This class includes the crucial GHG mitigation-cost function and the climate-damage function.

In the uncertainty runs, we have for each variable used a discrete distribution in which each variable takes five alternative values, each of which has a 20 percent probability. The interpretation is that the value for each cell is the mean of the distribution over the quintile of that variable's distribution. For normal variables, the quintiles represent the 12th, 32nd, 50th, 68th, and 88th percentile in the distribution. Alternatively, for normally distributed variables, the quintiles can be calculated as the mean plus -1.4, -0.53, 0.0, +0.53, and +1.4 normal standard errors. We sometimes refer to the "quintile range," which is the range between the top and bottom quintile or approximately between the 10th and 90th percentile.

1. GHG-temperature sensitivity coefficient (λ)

The equations used in the DICE model have been derived from a specification used in a small climate model and then calibrated to two larger general circulation models; the equations were also estimated using historical data on global mean temperatures for the last century. There is a marked disagreement both among the models and between the models and the historical data, although the statistical estimates using the historical data can be biased if there are slowly moving trend variables that are correlated with the greenhouse-gas signal.

The first probabilistic assessment of the value of the GHG-temperature sensitivity coefficient (T_{2xCO2}) dates back over a decade to the Charney report, which wrote, "We estimate that the most probable global warming for a doubling of CO_2 to be near 3° C with a probable error of ± 1.5° C. Our estimate is based primarily on our review of a series of calculations with three-dimensional models of the global atmospheric circulation..."[1] The meaning of the term "probable error" in the Charney Report has never been clear.

[1] See NRC [1979], p. 2.

For the analysis undertaken in this study, we proceed under the interpretation that the uncertainty range of 1.5 to 4.5° C should be interpreted as the central estimate minus and plus two standard deviations, which would be a conventional 95 percent confidence interval for normally distributed variables. This would imply a standard deviation of the temperature-sensitivity coefficient of .75° C. To allow for overconfidence (in line with the adjustments proposed in Nordhaus and Yohe [1983] and discussed later in this section) and to reflect the wide range of model results, we increases the standard deviation to 1.06° C.

2. GHG atmospheric retention rate (β)

The estimates of the uncertainty for the marginal atmospheric retention rate (β) are drawn primarily from the statistical estimates underlying the DICE model and presented in Chapter 3 of Nordhaus [1994]. In that chapter, we estimated the following model for CO_2 emissions and concentrations:

(D.1) $M(t) = [1-(1/\xi_M)]\, M(t-1) + \beta E(t-1)$

where M(t) is the deviation of CO_2 concentrations from its pre-industrial equilibrium, E(t) is anthropogenic CO_2 emissions, ξ_M is the turnover time of atmospheric CO_2, and β is the marginal atmospheric retention rate. The time period was for 1860 to 1989. The estimated coefficient was 0.64 with a standard error of 0.015. However, the coefficient was sensitive to the time period of estimation, with a low of 0.55 for the estimate of β using recursive least squares.

An alternative estimate of uncertainty was developed in the Nordhaus-Yohe study [1983], which used the top-bottom range for β of 0.10. Because of possible saturation effects and other non-linearities, we assume that the range of estimates from the recursive least squares estimates provide a better estimate of uncertainty than does the sample standard error of β. We therefore take a judgmental standard error of 0.10 for β. This yields a quintile range of 0.28.

3. Rate of decline of GHG-output ratio (g_σ)

The growth rate in the ratio of uncontrolled GHGs to world output, g_σ, is one of the important but generally overlooked uncertainties. Data prepared for the DICE model showed significant differences in the trend of the CO_2-GNP ratio in different countries and different time periods (see Chapter 4 of Nordhaus [1994]). Moreover, different energy models treat this parameter differently.

The trend in advanced industrial countries shows a generally declining trend in the CO_2-GNP ratio for the last six decades, with decline rates ranging from 0.9 to 1.8 percent per year for different subperiods. However, low-income and centrally planned countries show a rising trend in the CO_2-GNP ratio over the postwar period. Another approach is to use surveys of experts as an index of uncertainty. The 1991 International Energy Workshop poll of forecasts showed a two-standard deviation range of 0.6 percent per annum in the growth of the CO_2-GNP ratio for the period 1990-2020. Edmonds et al. [1986] used a closely related parameter -- the exogenous end-use energy efficiency -- which they estimated to occur at a rate of 1.3 percent per annum with a standard deviation of 0.7 percent per annum.

The variety of results suggest that a quintile range of approximately 2.5 percent per year in g_σ seems consistent with historical data, differences in model results, and the Edmonds et al. study. This assumption gives a one-in-five chance of no improvement in the CO_2-output ratio in the coming years.

4. Population growth (δ_L)

For both population growth and productivity growth, we adopt the uncertainty ranges developed by Nordhaus and Yohe [1983]. These ranges were developed for the 1983 National Academy study on "the CO_2 problem," were carefully reviewed by the committee and Academy, and have not been superseded in subsequent work. The approach used in that study was to view the dispersion in the results of different models or scientific studies as a reflection of the underlying uncertainty about the variable under study. The differences in opinion or of study results will usually measure the extent of scientific dissensus at a point of time. There are biases that may tend to either broaden or narrow the range of expert opinion, but this measure is easily replicated and can then be compared with more subjective measures.

For this and the next variable, we use the difference between the high and low values in Nordhaus-Yohe to provide an estimate of the range of values. According to that study, the difference between the high and low (the "high-low range") is very close to what we label the "quintile range," which is the difference between the top and bottom quintile in this study. Nordhaus-Yohe estimate the high-low range of the decline rate for population growth to be 2.1 percent per annum over the period 1965-2012. Other periods have slightly different estimates of the high-low range, but this value will be used in what follows. A study by Edmonds et al. [1986] used a similar methodology to that of Nordhaus and Yohe and prepared estimates of the uncertainty of a number of variables on the basis of both published estimates and interviews with experts.[2] Their estimates of the uncertainty of population growth was quite close to that found in the Nordhaus-Yohe study.

5. Productivity growth (δ_A)

The estimates for productivity growth follow the same methodology as those for population growth. The decline rate for productivity growth estimated in Nordhaus-Yohe was 1.3 percent per annum for the first four decades. The study by Edmonds et al. also derived estimates

[2] Edmonds et al. [1986].

of future productivity growth.[3] Their subjective standard deviations of productivity growth rates were 1.0 percent per annum for developed countries and 1.6 percent per annum for developing countries, as compared to 0.7 percent per annum for the Nordhaus-Yohe long-term estimates for the global economy.

For the current projections, we take the estimate of 2.0 percent per annum as the quintile range of the decline rate for productivity growth. This estimate is slightly higher than the Nordhaus-Yohe estimate to reflect the finding of the Edmonds et al. study. For the lowest quintile of projections of the deceleration of productivity growth, this implies that the current rate of total factor productivity growth of 1.1 percent per annum will decline very little in the next two centuries, while for the highest quintile of outcomes the current rate of productivity growth is assumed to halve every 30 years.

6. GHG mitigation cost function (b_1)

There are numerous estimates, particularly for CO_2, of the cost of reducing GHGs (see Nordhaus [1994] for a brief discussion). The DICE model assumes that the cost of reduction takes the following form:

(D.2) $TC(t) = Q(t)\ b_1\ \mu(t)^{b2}$

where $\mu(t)$ is the emissions control rate, $TC(t)$ is the total cost of the reduction, $Q(t)$ is output, and b_1 and b_2 represent the intercept and exponent of the cost function. In the uncertainty analysis, we consider the intercept only because of the difficulty of estimating various values of the exponent.

The range of estimates can be seen by comparing the estimates of the cost function for different models. In the Nordhaus survey [1991], the high-cost study has a cost about twice the best guess while the low cost study has a cost of about forty percent of the best guess. Cline's survey shows a range of 0.8 to 4.2 percent of GNP for a 50 percent reduction rate.[4] For emissions reduction rates up to 50 percent, the OECD survey of four models shows a range from high to low of a factor of about two for the United States and around four for China.[5]

Extracting a probability range from the surveys raises the same issues as for the climate models. We can here apply the methodology of Nordhaus and Yohe by assuming that the estimates of the cost function represent independent draws from the true distribution. For a

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[3] Edmonds et al. [1986].

[4] Cline [1992], p. 184.

[5] Hoeller et al. [1992], Figure 2.

uniform distribution, the range of the observations is between 2½ and 3 times the standard deviation for between 4 and 10 observations, whereas the range between the top and bottom quintiles is 2.8 standard deviations. This suggests that a range from high to low of a factor of 4 is consistent with the model comparisons. We then adjust this range up to a factor of five to correct for the tendency to underestimate the extent of uncertainty.

7. Climate-Damage Function (θ_1)

The equation for the relationship between global-temperature increase and income loss in the DICE model is given by:

(D.3) $$D(t) = Q(t)\theta_1 T(t)^{\theta 2}$$

(see equation (B.10) in section B above). In this equation, D(t) is the loss of global output, Q(t) is output, θ_1 is the intercept representing the scale of damage, and θ_2 is an exponent that represents the nonlinearity in the damage function. We parameterize the uncertainty about the damage function in terms of the intercept of the function, θ_1, for the same reasons as for the mitigation-cost function in the last section.

By comparison with other sectors of the model, there are few studies on which to base estimates of the uncertainty concerning the damages from climate change. See Cline [1992], Nordhaus [1994], and Fankhauser [1995] for a discussion. Because the three surveys just mentioned rely primarily on the same set of underlying studies, they do not seem a reliable basis for determining the uncertainty about the impacts of climate change using the Nordhaus-Yohe methodology. To fill in the gap, the author undertook a survey of experts on the economic impacts of climate change. For this section, we will use the results of a survey of experts (see Nordhaus [1994a]).

To construct the quintiles of the damage from the survey, we assume that the lowest quintile shows no damage, and that the median equals the mean; this leaves two free parameters. To close the distribution, we place the highest quintiles in such a way as to best approximate the ratio of the 90th percentile to the 50th percentile in the survey.

We have examined a number of alternative ways of estimating the distribution for climate damage, but all modifications of the distribution that stay within the conventions used for constructing the uncertain distributions make little difference to the outcome. Nonetheless, it must be emphasized that the estimates of the climate-damage function are extremely speculative.

8. Pure rate of time preference (ρ)

The issues involved in choosing the pure rate of time preference are deep but thoroughly discussed in the economic literature (see especially Lind [19??]). In general, the DICE model calibrates the pure rate of time preference from the equilibrium condition on the real return on capital given by $r(t) = \alpha g(t) + \rho$, where r(t) = real interest rate or the discount rate on goods, α is the elasticity of the marginal social utility of consumption, g(t) is the growth rate of per capita consumption, and ρ is the pure rate of social time preference.

For a number reasons, it proved difficult to include uncertainty about the rate of time preference in the estimates of the value of information. One reason is that this parameter concerns uncertainty about preferences rather than about technology, where the latter is the source of uncertainty of the other parameters. More important is the fact that when this parameter is included, it tends to put virtually all the weight on the state of the world with low time preference (because the future states are only slightly discounted for low discount rates). Given the difficulties in interpreting the uncertainty about the rate of time preference, this parameter has been excluded from experiments 2 through 5.

E. Latin Hypercube Sampling Procedures

This section explains the stratification used in the Latin hypercube sampling for the PRICE model. In general, we separate variables into those which induce the greatest dispersion in outcomes. For those with greatest dispersion -- which are productivity growth (δ_A), population growth (δ_A), and the rate of time preference (ρ) where that is used -- we use complete enumeration. This produces 125 possible combinations of these three parameters. For the remainder of the parameters, we employ Latin hypercube sampling coupled with the 125 combinations of the three completely enumerated parameters. The result is a total of 625 scenarios. Using Latin hypercube sampling on the remaining parameters means that for each of these parameters, each state of the world is represented *once and only once* for each combination of completely enumerated parameters. This section shows the stratification for each of the five experiments.

Experiment 1. Distribution of outcomes ("learn then act")

In the Monte Carlo simulations, there are three completely enumerated variables and five variables subject to Latin hypercube sampling. In this case, ten of the 625 runs might be the following:

δ_A	δ_L	ρ	g_σ	θ_1	λ	b_1	β
1	1	1	5	4	5	1	2
1	1	1	3	2	2	2	5
1	1	1	1	1	1	5	3
1	1	1	2	5	3	4	4
1	1	1	4	3	4	3	1
1	1	2	3	2	4	3	1
1	1	2	1	3	5	5	4
1	1	2	2	1	3	1	5
1	1	2	5	5	1	2	2
1	1	2	4	4	2	4	3

In this list, the numbers represent the state of the world of the parameter for that run. The selection of the five variables on the right are selected randomly without replacement. Note that for each random parameter, the SOW appears exactly once for each combination of the first three variables.

Experiment 2. Decisions with uncertainty ("act then learn")

For experiment 2, there is no sampling. Each SOW is selected to be a representative scenario as described in section C of this appendix.

Experiment 3. Act then learn with random parameters

For experiment 3, optimal policy with random parameter, 200 sets of five states of the world were randomly selected. Latin hypercube sampling was used again, so that each value of each parameter is used once and only once in each set of five. Seven parameters are used because the rate of time preference is held at 0.03 for all trials. One of the 200 sets of SOWs might be:

SOW	δ_A	δ_L	g_σ	θ_1	λ	b_1	β
1	3	2	4	1	1	4	5
2	5	1	3	4	5	2	3
3	1	3	2	5	4	3	1
4	4	5	1	2	2	5	2
5	2	4	5	3	3	1	4

Experiment 4. Value of Information with Single Variable: Expected Value Approach

For experiment 4, in which we obtain the value of information for individual parameters, the values of the non-target parameters are set equal to their expected values (also equal to their median values). Note that the discount rate is held at 0.03 for all trials. For example, if we wanted to learn the value of information for δ_A, the parameter values used in the PRICE model would be:

SOW	δ_A	δ_L	g_σ	θ_1	λ	b_1	β
1	1	3	3	3	3	3	3
2	2	3	3	3	3	3	3
3	3	3	3	3	3	3	3
4	4	3	3	3	3	3	3
5	5	3	3	3	3	3	3

Experiment 5. Value of Information with Single Variable: Random Value Approach

In the second experiment to value information for individual parameters, 100 sets of random values are chosen for each parameter. For each trial, this is the value that the parameter would take if the other parameters are known with certainty but are not necessarily at their median values. Note again that the discount rate is set at 0.03 for all trials. For example, to value δ_A, one of the 100 trials might be:

SOW	δ_A	δ_L	g_σ	θ_1	λ	b_1	β
1	1	5	2	5	4	1	3
2	2	5	2	5	4	1	3
3	3	5	2	5	4	1	3
4	4	5	2	5	4	1	3
5	5	5	2	5	4	1	3

Figure 1

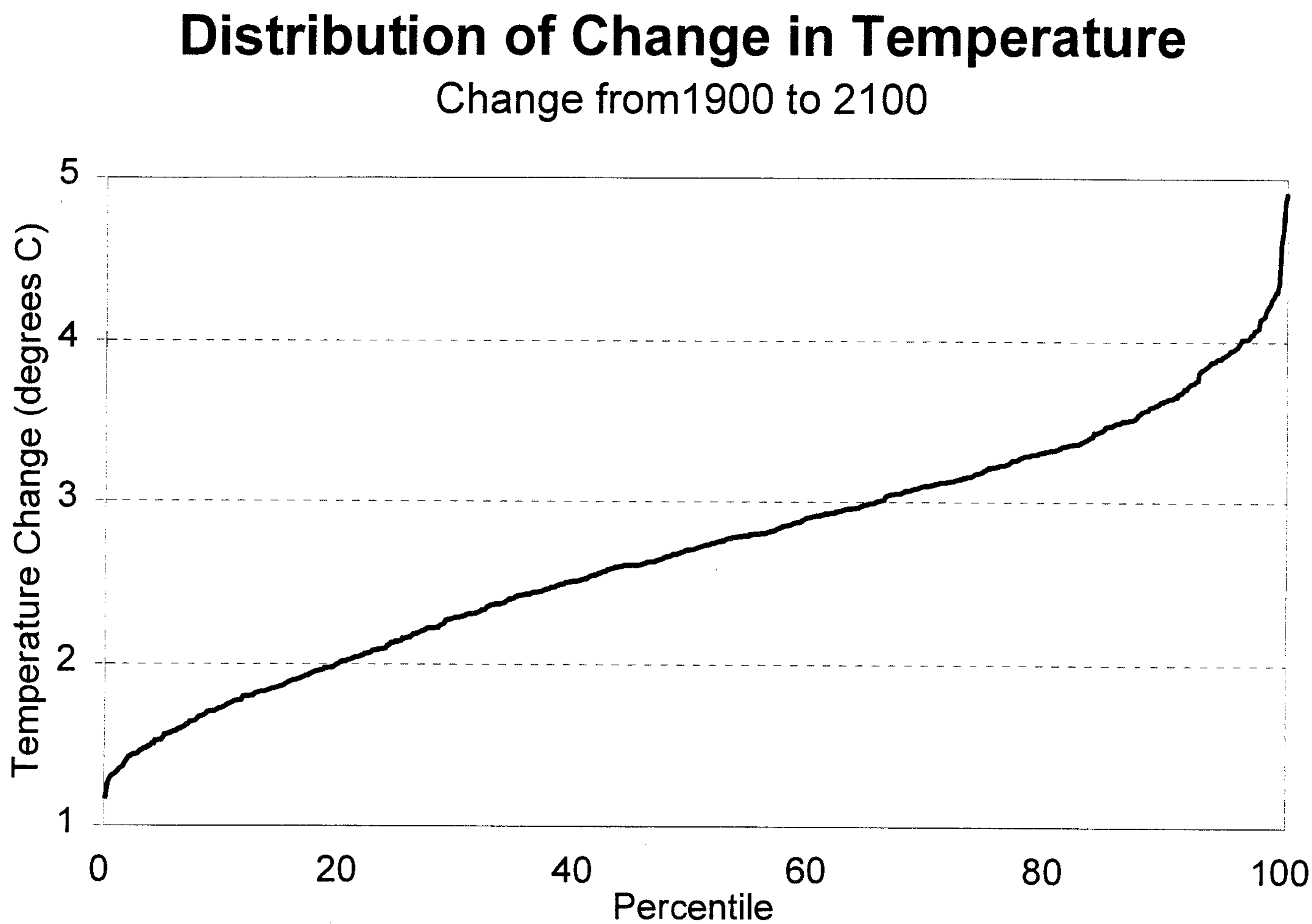


Figure 1 shows the distribution of the change in temperature from 1900 to 2100 from the Monte Carlo simulations of the PRICE model in a "learn, then act" approach.

Figure 2

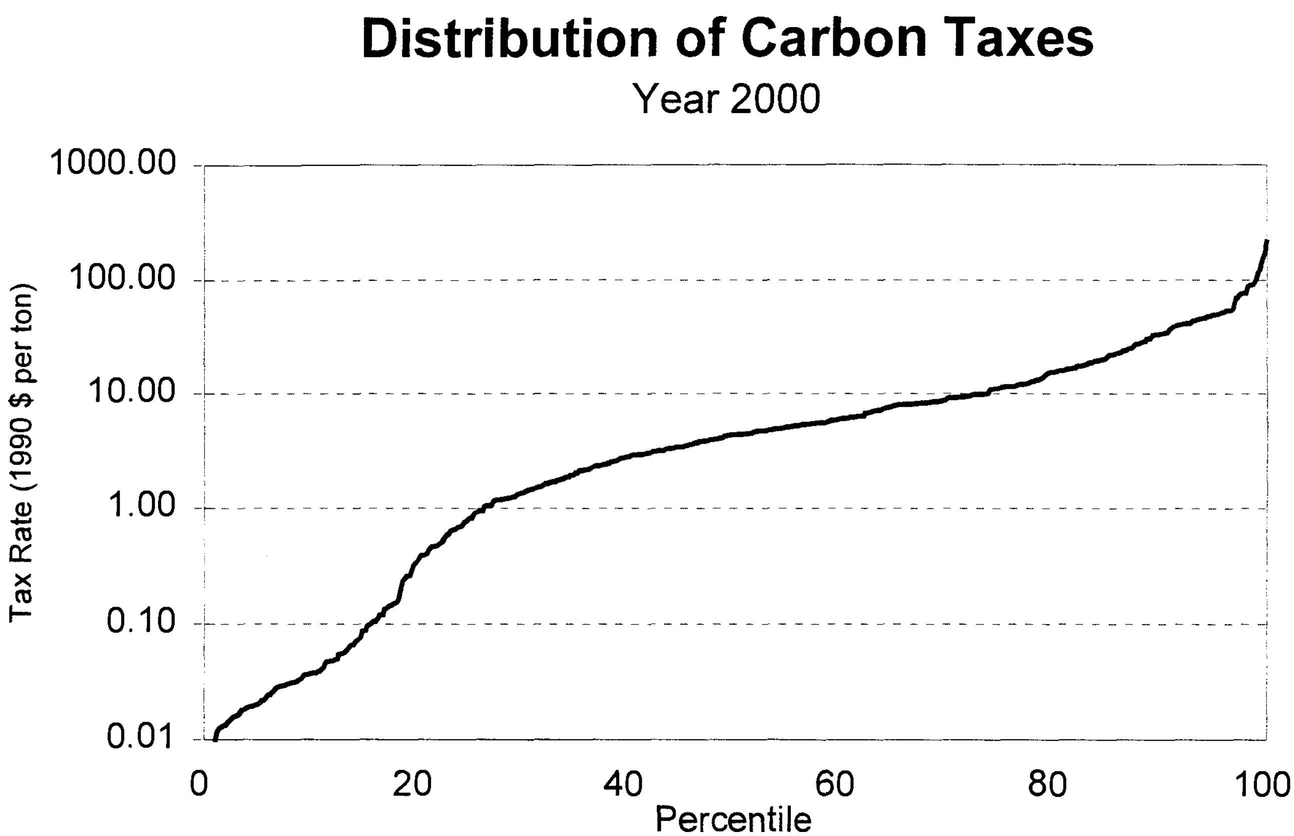


Figure 2 shows the distribution of efficient carbon taxes from the Monte Carlo simulations of the PRICE model in a "learn, then act" approach. Note that the scale of the y-axis is logarithmic.

Figure 3

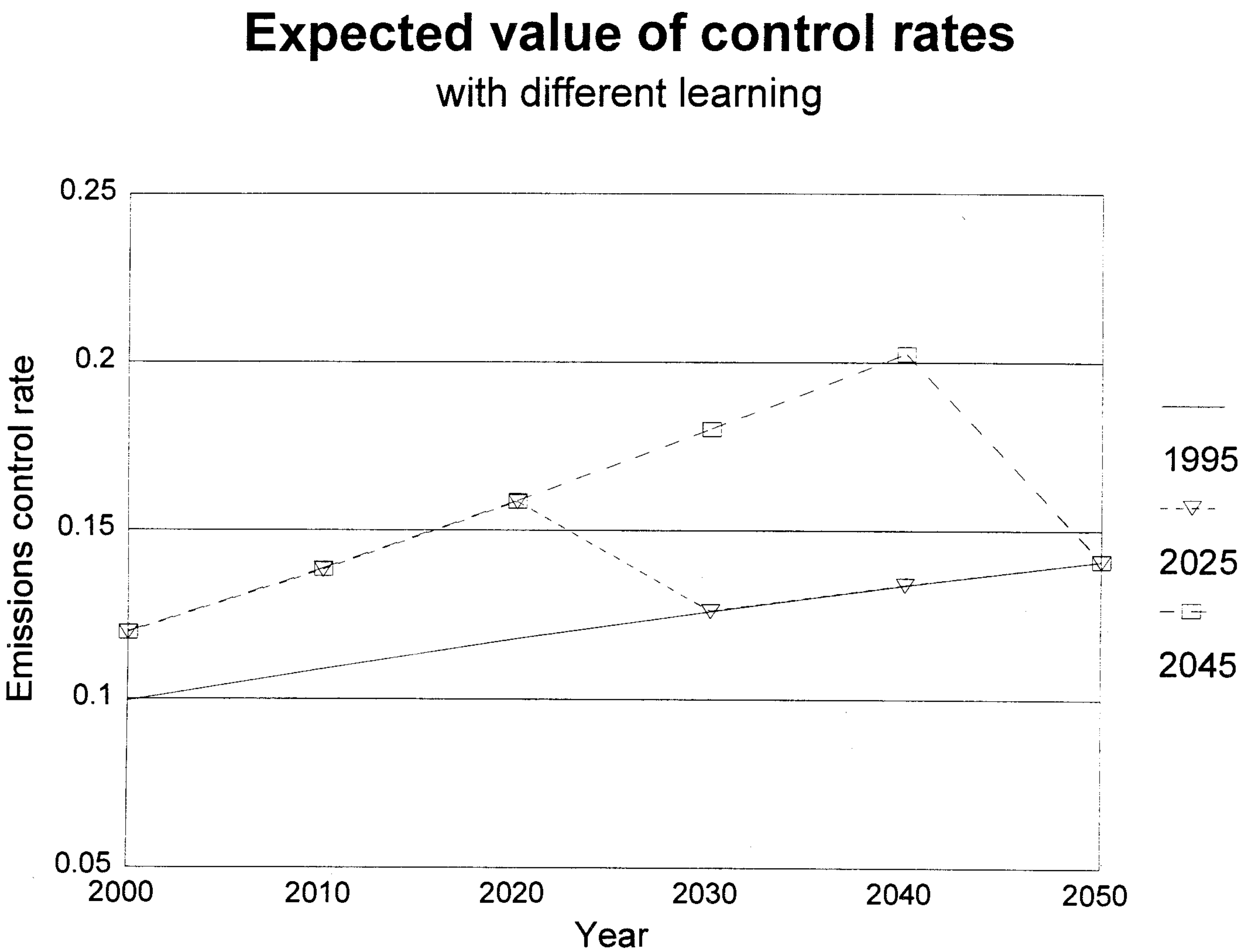
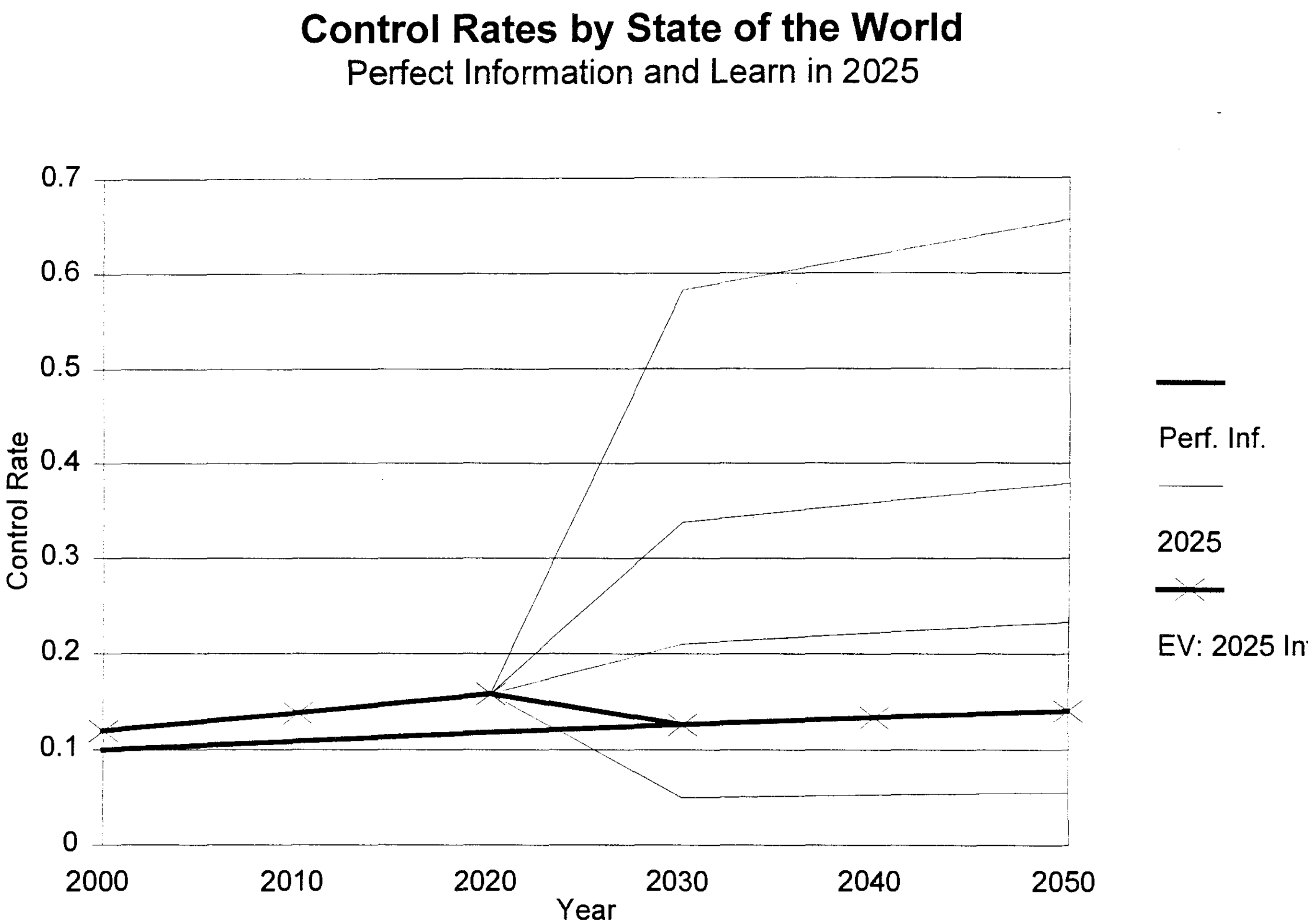


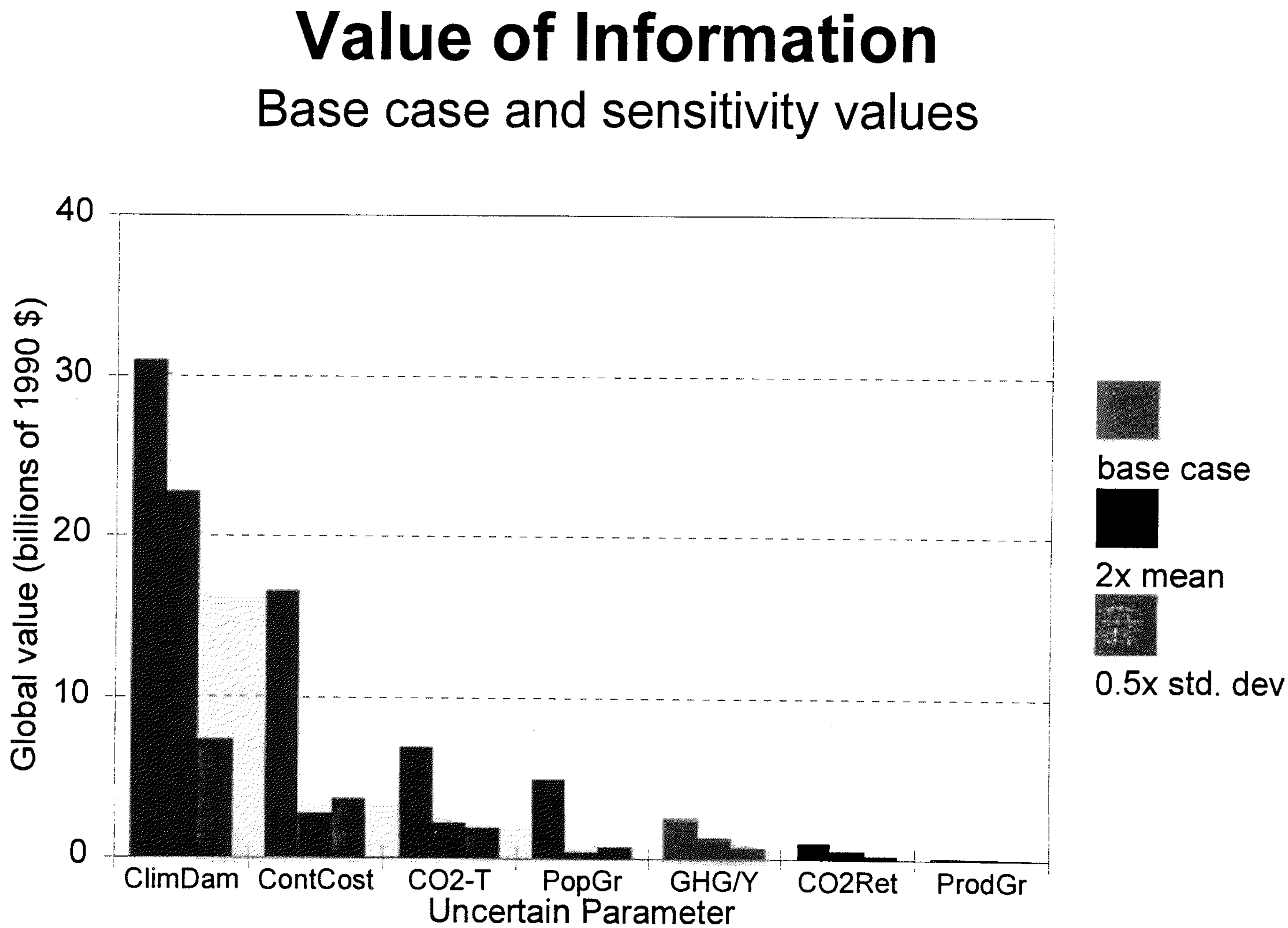
Figure 3 shows the *expected value* of the emissions control rate for different years of resolution of uncertainty. The top lines show that the expected value is higher before uncertainties are resolved, while the bottom lines are the expected values after resolution of uncertainty. Note that in the last period (2050), the expected value of the control rate is virtually identical whether uncertainty has been revealed in 1995 or 2045.

Figure 4



Thin lines show the emissions control rates in 5 SOWs before and after obtaining perfect information about the true state of the world. Before 2025, policies cannot be state-contingent, so control rates are equal in all SOWs; after 2025, policies can be state-contingent. Thick lines show the expected value of control rates for perfect information and with learning in 2025 (marked with x's). The solid lines show that the post-2025 average control rates is essentially unaffected by timing of information, but the expected values are higher with ignorance.

Figure 5



Bars show the global value of information for seven uncertain variables. These are, from left to right: damages from climate change (θ_1), cost of reducing CO_2 emissions (b_1), climate-CO2 coefficient (λ), population growth (δ_L), retention rate of CO_2 in the atmosphere (β), growth of GHG emissions-output ratio (g_σ), and productivity growth (δ_A). The left bar shows the base estimate of the value of information, the second bar shows the effect of doubling the mean of the uncertain parameter, and the third bar shows the effect of halving the standard deviation of the uncertainty. All figures are the present value of increased utility of consumption discounted to 1990 in 1990 prices.

Figure 6

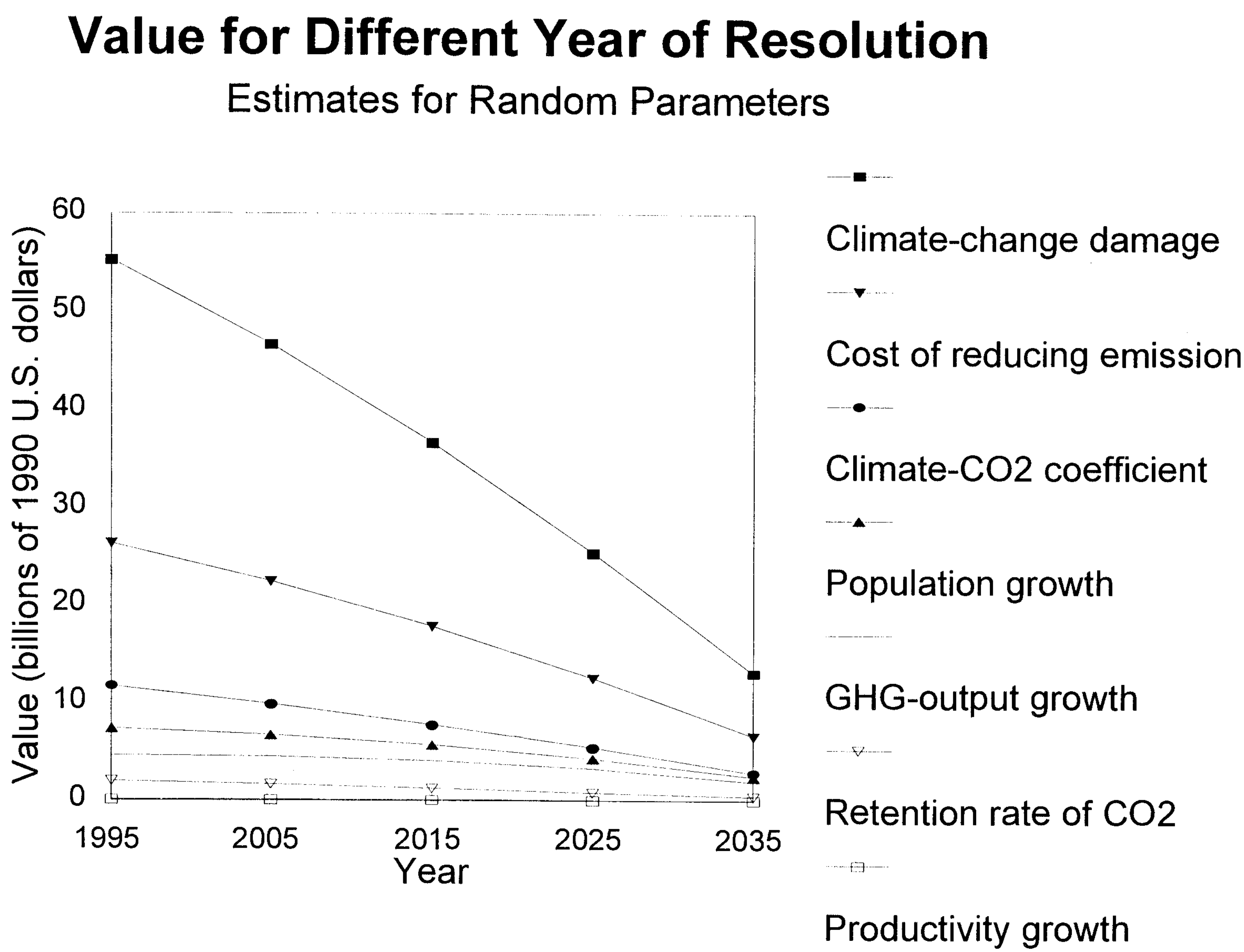


Figure 6 shows the value of information for each variable depending upon the year in which uncertainties are revealed. In each case, the value for that year is relative to the case where perfect information is attained in 2045.

Figure 7

Distribution of Value of Information

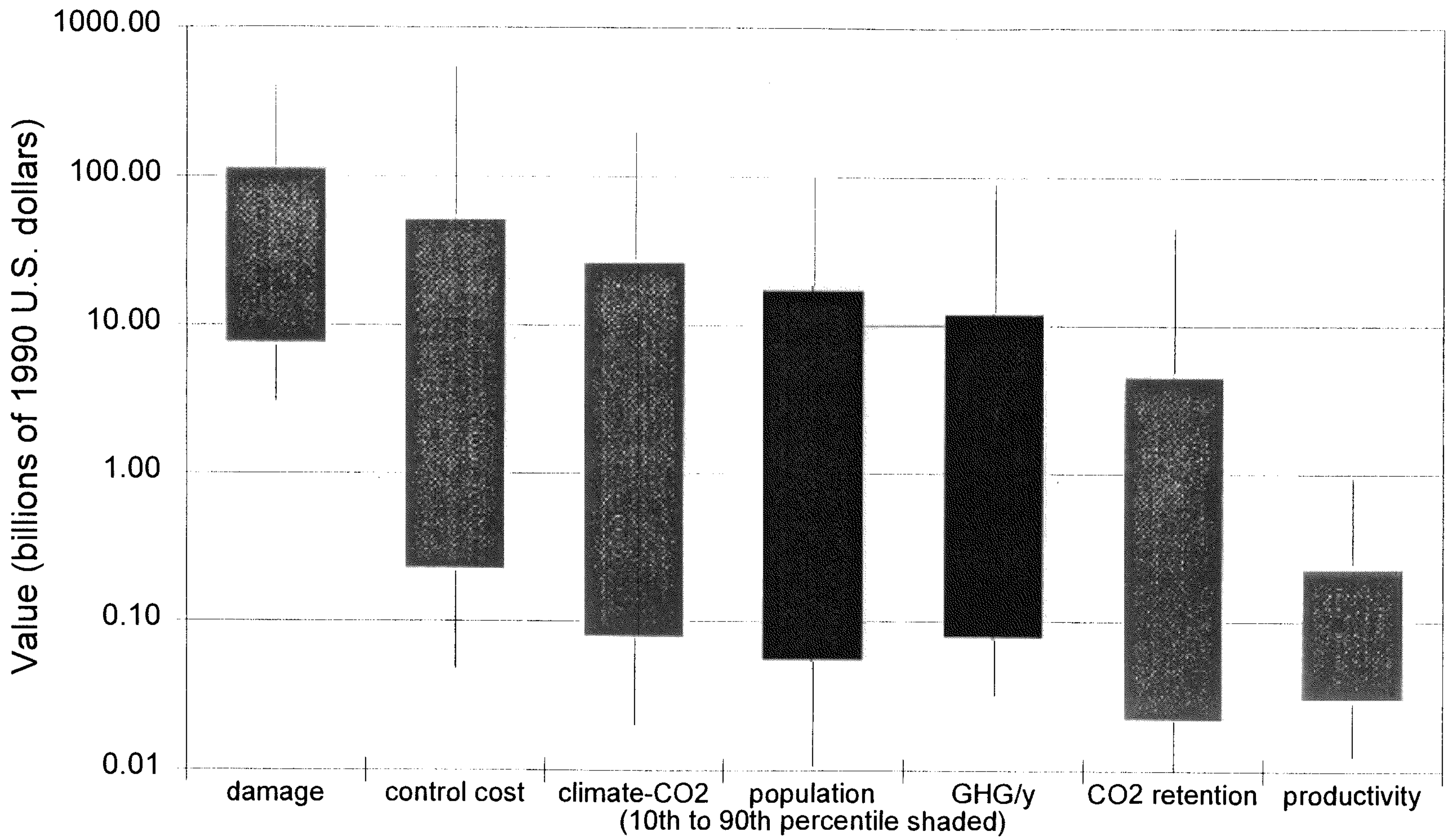


Figure 7 shows the distribution of the value of information calculated in experiment 5 (the value of information with single variable: random value approach). Lines represent the range of values calculated for each parameter. The figure considers the value of learning information now (1995) versus learning in 2045. The shaded bars encompass the values from the 10th to 90th percentile. It is interesting to note that the highest value of each distribution is well above the 90th percentile, indicating that particularly bad draws of the other states of the world can lead to a very large value of learning for a given parameter. The y-axis presents the value of information in billions of 1990 U.S. dollars. The values are the present value of increased utility of consumption discounted to 1990. Note that the scale is logarithmic.

Figure 8

Value of Information: all parameters

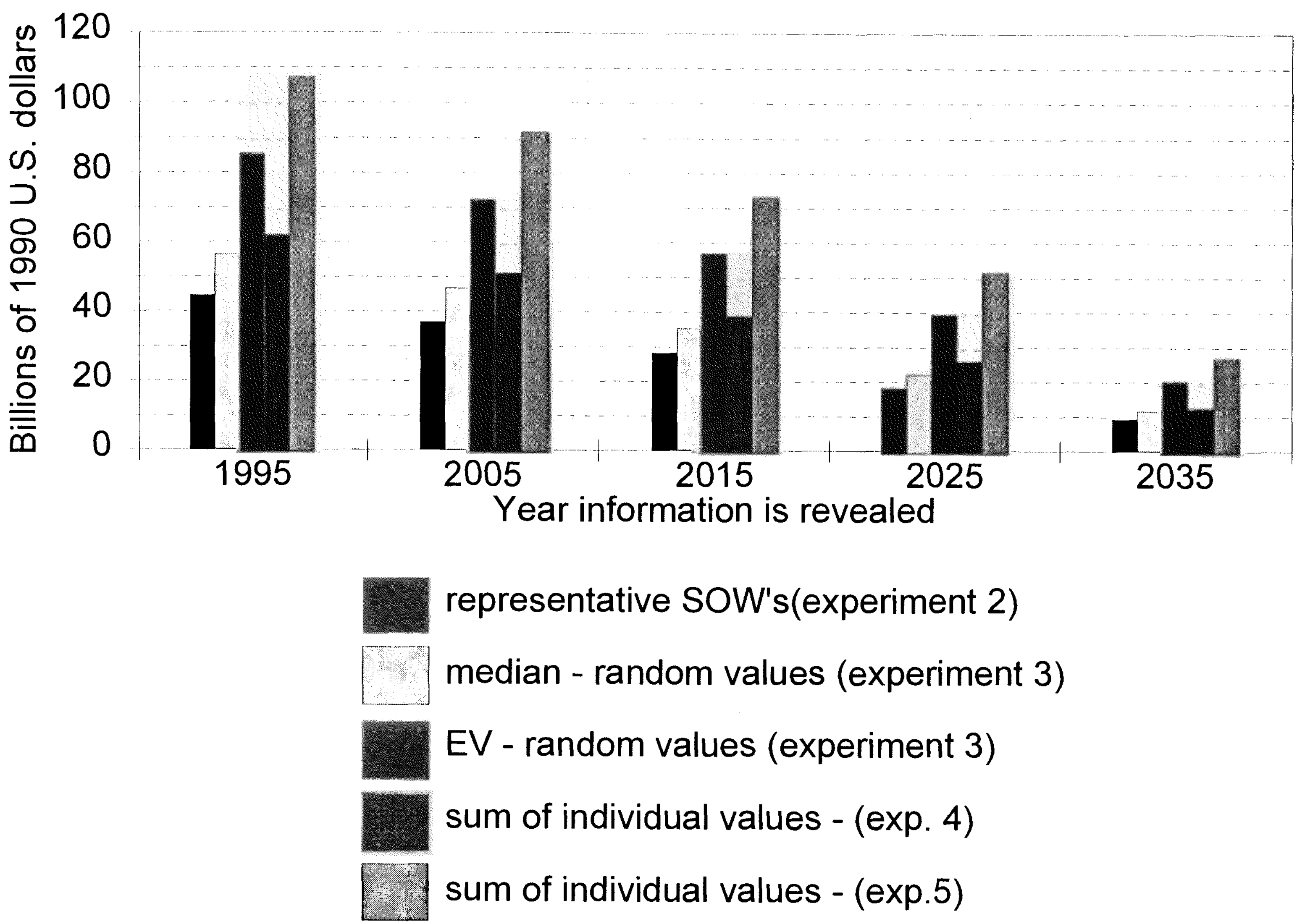


Table 1

Impact of Uncertainty on Carbon Tax and Control Rate

		CARBON TAX *Perfect Information*						CONTROL RATE *Perfect Information*					
SOW	Probability	2000	2010	2020	2030	2040	2050	2000	2010	2020	2030	2040	2050
1	0.02	$44.26	$60.36	$79.85	$102.81	$129.30	$159.32	0.458	0.498	0.537	0.576	0.614	0.652
2	0.08	$23.13	$31.79	$42.26	$54.50	$68.44	$83.89	0.266	0.290	0.314	0.337	0.358	0.379
3	0.15	$11.39	$15.84	$21.18	$27.33	$34.16	$41.49	0.168	0.183	0.197	0.210	0.222	0.233
4	0.25	$4.82	$6.83	$9.27	$12.09	$15.21	$18.55	0.097	0.107	0.116	0.125	0.132	0.139
5	0.50	$1.21	$1.72	$2.36	$3.10	$3.94	$4.84	0.039	0.043	0.047	0.050	0.053	0.056
Average		$6.25	$8.69	$11.65	$15.09	$18.96	$23.18	0.100	0.109	0.118	0.126	0.134	0.141

		CARBON TAX *Learn in 2025*						CONTROL RATE *Learn in 2025*					
SOW	Probability	2000	2010	2020	2030	2040	2050	2000	2010	2020	2030	2040	2050
1	0.02	$6.31	$8.78	$11.74	$105.47	$131.99	$161.89	0.120	0.138	0.158	0.582	0.620	0.657
2	0.08	$6.31	$8.78	$11.74	$54.91	$68.84	$84.23	0.120	0.138	0.158	0.338	0.359	0.379
3	0.15	$6.31	$8.78	$11.74	$27.40	$34.22	$41.54	0.120	0.138	0.158	0.210	0.222	0.233
4	0.25	$6.31	$8.78	$11.74	$12.06	$15.19	$18.53	0.120	0.138	0.158	0.125	0.132	0.139
5	0.50	$6.31	$8.78	$11.74	$3.08	$3.92	$4.82	0.120	0.138	0.158	0.050	0.053	0.056
Average		$6.31	$8.78	$11.74	$15.17	$19.03	$23.25	0.120	0.138	0.158	0.126	0.134	0.141

		CARBON TAX *Learn in 2045*						CONTROL RATE *Learn in 2045*					
SOW	Probability	2000	2010	2020	2030	2040	2050	2000	2010	2020	2030	2040	2050
1	0.02	$6.34	$8.81	$11.79	$15.25	$19.12	$163.89	0.120	0.139	0.159	0.180	0.203	0.660
2	0.08	$6.34	$8.81	$11.79	$15.25	$19.12	$84.31	0.120	0.139	0.159	0.180	0.203	0.379
3	0.15	$6.34	$8.81	$11.79	$15.25	$19.12	$41.55	0.120	0.139	0.159	0.180	0.203	0.233
4	0.25	$6.34	$8.81	$11.79	$15.25	$19.12	$18.52	0.120	0.139	0.159	0.180	0.203	0.139
5	0.50	$6.34	$8.81	$11.79	$15.25	$19.12	$4.81	0.120	0.139	0.159	0.180	0.203	0.056
Average		$6.34	$8.81	$11.79	$15.25	$19.12	$23.29	0.120	0.139	0.159	0.180	0.203	0.141

Table 1 shows how the carbon tax and emissions control rate vary as the year of learning is varied in experiment 2: decisions with uncertainty. Values for each state of the world are presented for three posible learning scenarios: perfect information, learning in 2025, and learning in 2045. In years before learning occurs, the optimal policy is constrained to be the same across all states of the world.

Table 2

Value of Early Information in Experiment 2: Decisions with Uncertainty
Value of Early Information vs. Information in 2045
(billions of 1990 U.S. dollars)

Year information is revealed:	Value of Information
Perfect information	44.7
2005 information	37.0
2015 information	28.3
2025 information	18.9
2035 information	9.3
2045 information	0.0

Table 2 shows how the value of information changes as the year in which information is revealed changes for experiment 2 (decisions with uncertainty). All values are in billions of 1990 U.S. dollars, and represent the present value increased utility of consumption discounted to 1990.

Table 3

Value of Learning in Experiment 3: Act then Learn with Random Parameters
Value of Early Informatin vs. Information in 2045
(billions of 1990 U.S. dollars)

Year information is revealed:	Expected Value	*percentile:* 10th	50th	90th
Perfect information (1995)	85.80	19.03	56.46	159.02
	(8.06)	(1.78)	(4.96)	(22.51)
2005 information	72.80	14.88	46.83	140.41
	(6.24)	(1.68)	(4.94)	(20.85)
2015 information	57.40	10.66	35.22	115.62
	(5.45)	(0.99)	(3.79)	(20.61)
2025 information	39.98	7.19	22.89	80.82
	(3.95)	(0.80)	(2.84)	(13.59)
2035 information	20.71	3.49	11.59	41.76
	(2.03)	(0.41)	(1.20)	(6.92)

N = 200

Table 3 presents the expected value of information from the 200 runs of experiment 3 (act then learn with random parameters). Values are presented for each of the possible years in which information is revealed. In addition, the 10th, 50th, and 90th percentile are presented. Note that the distributions are skewed toward high values of information, as the expected value is significantly larger than the median for each run. All values represent the gain in the present value of increased utility of consumption which results from learning in the given year, rather than learning in 2045.

Table 4

Value of Information of Single Variables
Experiment 4: Expected Value Approach
Value of Early Information vs. Information in 2045
(billions of 1990 U.S. dollars)

I. Base Case - unknown parameters same as in previous experiments

	Year information is revealed:		
Uncertain parameter	**1995**	**2015**	**2035**
Cost of climate change	30.89	18.58	5.76
Mitigation cost	16.48	10.24	3.35
Climate feedback	6.91	4.27	1.38
Population growth decline	4.84	3.69	1.53
Decline of GHG-output rate	2.40	2.08	0.92
GHG retention rate	1.00	0.56	0.16
Productivity growth decline	0.09	0.09	0.06

II. Expected Value Doubled/Standard Deviation Same as Base Case

	Year information is revealed:					
	1995		**2015**		**2035**	
Uncertain parameter	*Value*	*Ratio*	*Value*	*Ratio*	*Value*	*Ratio*
Cost of climate change	22.690	0.735	14.058	0.757	4.535	0.787
Mitigation cost	2.680	0.163	1.720	0.168	0.587	0.175
Climate feedback	2.225	0.322	1.441	0.338	0.499	0.360
decline of GHG-output rate	1.251	0.258	1.039	0.282	0.426	0.279
GHG retention rate	0.525	0.219	0.254	0.122	0.056	0.061
Population growth decline	0.374	0.375	0.266	0.474	0.103	0.659
Productivity growth decline	0.077	0.874	0.075	0.858	0.044	0.784

III. Standard Deviation Half/Expected Value Same as Base Case

	Year information is revealed:					
	1995		**2015**		**2035**	
Uncertain parameter	*Value*	*Ratio*	*Value*	*Ratio*	*Value*	*Ratio*
Cost of climate change	7.350	0.238	4.399	0.237	1.352	0.235
Mitigation cost	3.612	0.219	2.263	0.221	0.749	0.224
Climate feedback	1.822	0.264	1.144	0.268	0.380	0.275
Population growth decline	0.682	0.141	0.505	0.137	0.201	0.132
decline of GHG-output rate	0.671	0.280	0.584	0.280	0.260	0.283
GHG retention rate	0.212	0.212	0.113	0.201	0.028	0.182
Productivity growth decline	0.039	0.442	0.038	0.434	0.023	0.416

Table 4 presents the value of information for each parameter found in experiment 4. In this experiment, one parameter is assumed uncertain. All other parameters are assumed to be known to be at their median values. Panel I presents the results for the base case, in which the valus for the five states of the world for the uncertain parameter is the same as in the other experiments. These values can be found in table A-1. The second panel presents the values when the expected value of the uncertain parameter is doubled. The standard deviation of the uncertain parameter is kept the same as in panel I, and the known parameters are still assumed to be at their median values. Finally, the third panel presents the values when the standard deviation of the uncertain parameter is halved, with the expected value remaining the same as in panel I. For the second and third panels, the ratio of the new values to the values from the base case run are presented as well.

Expected Value of Information of Single Variables
Experiment 5: Random Value Approach

Value of Early Information v. Information in 2045
(billions of 1990 U.S. dollars)

	Year information is revealed:		
Uncertain Parameter	**1995**	**2015**	**2035**
Cost of climate change	55.33	36.68	13.07
	(7.66)	(6.14)	(1.80)
Mitigation cost	26.34	17.85	6.63
	(6.93)	(4.92)	(2.62)
Climate feedback	11.69	7.81	2.84
	(2.97)	(1.91)	(0.75)
Population growth decline	7.40	5.71	2.44
	(1.43)	(1.24)	(0.57)
Decline of GHG-output rate	4.62	4.12	1.96
	(1.19)	(1.04)	(0.52)
GHG retention rate	2.02	1.29	0.44
	(0.54)	(0.43)	(0.15)
Productivity growth decline	0.12	0.11	0.07
	(0.02)	(0.01)	(0.01)

Note: Bootstrap standard errors in parenthesis
N=100

Table 5 presents the value of information for each parameter found in experiment 5. In this experiment, one parameter is assumend uncertain. The other parameters are assumed to be known, but the values at which they are known are considered random. 100 separate trials are conducted for each parameter. In each trial, a new set of known values was chosen for the other parameters. This table presents the expected value of information for each parameter, as well as the standard errors of the expected values. Standard errors were calculated via the bootstrap method.

Model	*Reference*	***Value of Information for All 7 Parameters (Perfect Information Relative to 2045, billions of 1990 $)***
PRICE model: all parameters uncertain:		
Average value[1]	Table 2	44.7
Random value[2]	Table 3	85.8 (8.1)
PRICE model: sum of individual uncertain parameters:		
Average value[3]	Table 4	62.61
Random value[4]	Table 5	107.62

Table 6. Summary Estimate of the Value of Information

Notes to table 6:

[1] Results from experiment 2: Decisions with uncertainty ("act then learn")

[2] Results from experiment 3: Act then learn with random parameters. Standard error in parenthesis.

[3] Results are the sum of the individual parameter values from experiment 4: Value of information with single variable: expected value approach.

[4] Results are the sum of the individual parameter values from experiment 5: Value of information with single variable: random value approach.

Table A1

I. Distribution of Uncertain Parameters

Base Case Parameter	exp. val.	median	std.dev.	*State of the World:* 1	2	3	4	5
Pop. growth decline	0.2000	0.2000	0.1138	0.0320	0.1320	0.2000	0.2730	0.3630
Prod. growth decline	0.1000	0.1000	0.0678	0.0200	0.0410	0.1000	0.1280	0.2110
GHG retention rate	0.6400	0.6400	0.0947	0.5000	0.5870	0.6400	0.6930	0.7800
Climate feedback	1.6245	1.4000	0.6679	0.9330	1.1200	1.4000	1.8667	2.8029
Climate sensitivity	2.9286	2.9286	1.0363	4.3944	3.6607	2.9286	2.1964	1.4628
Time preference	0.0300	0.0300	0.0141	0.0100	0.0200	0.0300	0.0400	0.0500
GHG-output rate	-0.1168	-0.1168	0.0764	-0.0112	-0.0628	-0.1168	-0.1620	-0.2312
Mitigation cost	0.0686	0.0686	0.0381	0.0270	0.0340	0.0686	0.0800	0.1334
Climate change	0.0133	0.0133	0.0115	0.0001	0.0040	0.0133	0.0160	0.0331

II. Distribution of Uncertain Parameters in Experiment 2 - Act then Learn

Parameter	exp. val.	median	std.dev.	*State of the World:* 1	2	3	4	5
Pop. growth decline	0.1999	0.1850	0.0236	0.078	0.185	0.170	0.218	0.207
Prod. growth decline	0.0997	0.0990	0.0014	0.102	0.098	0.099	0.102	0.099
GHG retention rate	0.6400	0.6670	0.0196	0.709	0.672	0.667	0.624	0.632
Climate feedback	1.6255	1.3040	0.2488	0.991	1.163	1.304	1.606	1.831
Climate sensitivity	2.5942	3.1442	0.4752	4.137	3.525	3.144	2.553	2.239
Time preference	0.0300	0.0300	0.0000	0.030	0.030	0.030	0.030	0.030
GHG-output rate	-0.1171	-0.1160	0.0055	-0.120	-0.101	-0.114	-0.116	-0.121
Mitigation cost	0.0685	0.0690	0.0030	0.076	0.075	0.063	0.068	0.069
Climate change	0.0133	0.0220	0.0092	0.033	0.032	0.022	0.017	0.005

The above tables present the values of the uncertain parameters for each of the five states of the world. Panel I gives the values for all experiments except experiment 2. The parameter values for experiment 2 are the"representative scenarios" derived by stratifying the results of the Monte Carlo sample in experiment 1. The values for the representative scenario are given in panel II.

Table A-2

Summary Statistics for Experiment 1 - Learn Then Act

Mean	**2000**	**2050**	**2100**
Output	31.014	93.496	196.730
	(0.00934)	(0.769)	(3.950)
Control Rates	0.104	0.150	0.184
	(0.00381)	(0.00591)	(0.00834)
Temperature	0.635	1.553	2.706
	(0.00268)	(0.0131)	(0.0260)
CO2 Emissions	89.261	174.252	278.523
	(0.453)	(2.892)	(8.660)
CO2 Concentrations	782.296	1052.978	1473.095
	(0.289)	(3.749)	(16.210)
Carbon Tax	$11.64	$34.71	$67.23
	(0.781)	(2.473)	(5.672)

Standard Deviation	**2000**	**2050**	**2100**
Output	0.225	20.674	107.494
	(0.00542)	(0.527)	(4.186)
Control Rates	0.101	0.154	0.198
	(0.00465)	(0.00781)	(0.0112)
Temperature	0.076	0.332	0.720
	(0.00164)	(0.00705)	(0.0175)
CO2 Emissions	11.800	70.701	210.038
	(0.457)	(2.191)	(12.571)
CO2 Concentrations	7.570	97.905	388.515
	(0.153)	(2.767)	(17.062)
Carbon Tax	$21.71	$67.50	$144.04
	(2.171)	(6.911)	(16.861)

Coeff. of Variation	**2000**	**2050**	**2100**
Output	0.007	0.221	0.546
	(0.000175)	(0.00535)	(0.0146)
Control Rates	0.972	1.022	1.077
	(0.0328)	(0.0356)	(0.0370)
Temperature	0.119	0.214	0.266
	(0.00296)	(0.00545)	(0.00628)
CO2 Emissions	0.132	0.406	0.754
	(0.00555)	(0.0101)	(0.0325)
CO2 Concentrations	0.010	0.093	0.264
	(0.000195)	(0.00260)	(0.00965)
Carbon Tax	1.865	1.944	2.143
	(0.113)	(0.125)	(0.145)

10th Percentile	**2000**	**2050**	**2100**
Output	30.718	69.623	94.046
	(0.00749)	(0.847)	(2.087)
Control Rates	0.010	0.010	0.012
	(0.000)	(0.000442)	(0.000823)
Temperature	0.501	1.063	1.734
	(0.000)	(0.00545)	(0.0372)
CO2 Emissions	73.669	95.203	91.014
	(1.112)	(2.654)	(4.019)
CO2 Concentrations	771.11	934.77	1060.66
	(0.000)	(4.600)	(10.074)
Carbon Tax	$0.04	$0.11	$0.18
	(0.00670)	(0.0135)	(0.0322)

Median	**2000**	**2050**	**2100**
Output	31.010	89.793	94.046
	(0.0123)	(0.899)	(3.278)
Control Rates	0.073	0.107	0.125
	(0.00371)	(0.00622)	(0.00672)
Temperature	0.660	1.590	2.709
	(0.000)	(0.0203)	(0.0390)
CO2 Emissions	90.179	163.546	219.622
	(0.558)	(3.927)	(8.606)
CO2 Concentrations	782.30	1038.26	1397.53
	(0.000)	(6.040)	(18.488)
Carbon Tax	$4.39	$11.90	$21.30
	(0.291)	(0.945)	(1.347)

90th Percentile	**2000**	**2050**	**2100**
Output	31.302	123.833	319.669
	(0.0139)	(1.876)	(14.359)
Control Rates	0.244	0.353	0.433
	(0.0109)	(0.0156)	(0.0268)
Temperature	0.713	1.962	3.645
	(0.000)	(0.0107)	(0.0452)
CO2 Emissions	103.688	274.172	540.159
	(0.455)	(8.655)	(26534)
CO2 Concentrations	793.48	1186.99	2002.04
	(0.000)	(8.102)	(43.959)
Carbon Tax	$33.85	$101.50	$177.50
	(3.634)	(11.004)	(21.649)

Note : standard errors in parenthesis
Number of sampling runs = 625

Table A-2 presents summary statistics for the Monte Carlo sample in experiment 1.